



DEPARTMENT OF EDUCATION

GRADE 11 GENERAL MATHEMATICS

11.1: NUMBERS AND APPLICATIONS



FODE DISTANCE LEARNING



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GRADE 11

GENERAL MATHEMATICS

MODULE 1

NUMBERS AND APPLICATIONS

TOPIC 1: BASIC NUMERACY

TOPIC 2: LAWS OF INDICES

TOPIC 3: SURDS

TOPIC 4: UNITS OF MEASUREMENTS



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DIANA TEIT AKIS

PRINCIPAL



Flexible Open and Distance Education
Papua New Guinea

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SECRETARY'S MESSAGE

Achieving a better future by individual students and their families, communities or the nation as a whole, depends on the kind of curriculum and the way it is delivered.

This course is a part of the new Flexible, Open and Distance Education curriculum. The learning outcomes are student-centred and allows for them to be demonstrated and assessed.

It maintains the rationale, goals, aims and principles of the national curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision by Flexible, Open and Distance Education as an alternative pathway of formal education.

The course promotes Papua New Guinea values and beliefs which are found in our Constitution, Government Policies and Reports. It is developed in line with the National Education Plan (2005 -2014) and addresses an increase in the number of school leavers affected by the lack of access into secondary and higher educational institutions.

Flexible, Open and Distance Education curriculum is guided by the Department of Education's Mission which is fivefold:

- To facilitate and promote the integral development of every individual
- To develop and encourage an education system satisfies the requirements of Papua New Guinea and its people
- To establish, preserve and improve standards of education throughout Papua New Guinea
- To make the benefits of such education available as widely as possible to all of the people
- To make the education accessible to the poor and physically, mentally and socially handicapped as well as to those who are educationally disadvantaged.

The college is enhanced to provide alternative and comparable pathways for students and adults to complete their education through a one system, many pathways and same outcomes.

It is our vision that Papua New Guineans' harness all appropriate and affordable technologies to pursue this program.

I commend all those teachers, curriculum writers, university lecturers and many others who have contributed in developing this course.

UKE KOMBRA, PhD
Secretary for Education



11.1 NUMBERS AND APPLICATIONS

Introduction

This Unit focuses on the mathematics used every day in our communities to measure, compare and present information numerically. There is an emphasis on the development of real numbers and their everyday usage in learning mathematics.

This is the first unit in the series of Mathematics B for Grade 11. As you go along with the unit, you will find lessons and activities which will help you make use of appropriate techniques and instruments to estimate and calculate physical quantities. In addition, this unit will also lead you to apply knowledge of numbers and their relationships to investigate a range of different context.

11.1.1: BASIC NUMERACY

This topic introduces real numbers in complex number field; and revises multiples and factors, operations with fractions, estimation and error, and writing digits correct to specified significant digits.

11.1.2: LAWS OF INDICES

This topic revises basic concepts of indices, and laws of indices. The topic provides the basis to study logarithms and exponential functions and graphs.

11.1.3: SURDS

This topic evolves from laws of indices; it expands on to application of basic operations on surds. It explains the approximation and exact values of quantities and measurements.

11.1.4: UNITS OF MEASUREMENTS

The topic covers Metric and Imperial Measurement. Then expands on to quadratic equations and graphs. It also discusses inequalities and systems of inequalities.

You will also find that most of the lessons in this unit are revisions and continuations of the lessons you have learnt in your lower secondary mathematics. This serves as a springboard for the more challenging and more complicated units. It also gives you the time to master the skills needed for higher Mathematics courses such as Trigonometry and Calculus.



LEARNING OUTCOMES

On successful completion of this module, you will be able to:

- discuss the historical development of real numbers
- classify and relate symbols to all real numbers
- plot real numbers on the real number line
- apply the properties of real numbers
- state the law or properties of surds.
- apply properties of surds
- simplify and rationalize surds
- state the number of significant figures
- round off significant figures
- apply the laws of indices
- express index numbers in surds form
- write metric measurements of length, mass and capacity
- convert metric measurement to imperial or vice versa
- apply scales on actual lengths on the ground



TIME FRAME

This unit should be completed within 10 weeks.

If you set an average of 3 hours per day, you should be able to complete the unit comfortably by the end of the assigned week.

Try to do all the learning activities and compare your answers with the ones provided at the end of the unit. If you do not get a particular exercise right in the first attempt, you should not get discouraged but instead, go back and attempt it again. If you still do not get it right after several attempts then you should seek help from your friend or even your tutor. Do not pass any question without solving it first.



11.1.1 Basic Numeracy

The term numeracy is similar to the word literacy. Just as the early definitions of the literacy have progressed from “reading and writing”, numeracy is more than “numbers and measurements”. In the eighties, the British Cockcroft Committee developed a definition of numeracy. Cockcroft Committee stated that a numerate person should understand some of the ways mathematics can be used for communication, and this required the possession of two attributes:

1. being “at-ease” with all those aspects of mathematics that enable a person to cope with the practical demands of everyday life, and
2. the ability to understand information presented in mathematical terms.

Basic numeracy skills consist of comprehending fundamental mathematics like addition, subtraction, multiplication, and division.

For example, if one can understand simple mathematical equations such as $2 + 2 = 4$, then one would be considered possessing at least basic numeric knowledge.

Substantial aspects of numeracy also include number sense, operation sense, computation, measurement, geometry, probability and statistics. A numerically literate person can manage and respond to the mathematical demands of life.

Basic Numeracy skills count. It is not just for teachers, scientists, accountants and engineers. Many professions require at least basic level of understanding when it comes to numeracy.

The numbers and their symbols that we use today have advanced over many years. People began to use mathematics in their lives when they first started to use numbers in counting objects.

Later on, they became farmers and builders and the system and the way of writing numbers became more sophisticated. Its adaptation in human civilization developed trade, science, arts, ownership, structures, insurance, technology, sports and sense of time.

We will use decimal system (base 10) in all our discussions. But there are also other base systems such as binary system (Base 2 System), quinary system (Base 5 System) and duodecimal system (Base 12 System).

Binary System

The Decimal System uses the numbers from 0 to 9, Quinary System uses the numbers 0, 1, 2, 3, 4 and Binary System uses the numbers 0 and 1 only. There are other base systems we seldom use in our daily mathematics.



They can be expressed as 1101_{10} , 1101_5 and 1101_2 . The digits are the same but the bases are different therefore, their values are different.

Now, compare the two numbers $1 \times 10^3 + 1 \times 10^2 + 1 \times 10^1 + 1 \times 10^0$ and $1 \times 10^3 + 0 \times 10^2 + 1 \times 10^1 + 1 \times 10^0$. The first can be both, a decimal or a binary number, but the second is a binary number.

The table below shows the first ten binary digits.

0	1	2	3	4	5	6	7	8	9
0	1	10	11	100	101	110	111	1000	1001

You can observe the pattern of placement of 0 and 1 to expand the table of equivalents to first 20 digits. Computer programming uses binary system.

Example Convert 63 to a binary number.

Solution

	Remainder	Power of 2	Explanation
63	1	0	$63 \div 2 = 21$ remainder 1
21	1	1	$21 \div 2 = 10$ remainder 1
10	0	2	$10 \div 2 = 5$ remainder 0
5	1	3	$5 \div 2 = 2$ remainder 1
2	0	4	$2 \div 2 = 1$ remainder 0
1	1	5	$1 \div 2 = 0$ remainder 1

Therefore, $63_{10} = 110101_2$

To convert binary number into a decimal number we use the table below. The table can be extended either way if there are more digits.

2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
64	32	16	8	4	2	1	0.5	0.25	0.125

Example Write the binary number 10111 in its decimal equivalent.

Solution

$$\begin{aligned} 10111_2 &= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ &= 16 + 0 + 4 + 2 + 1 \\ &= 23_{10} \end{aligned}$$

You can use the above examples to convert other base numbers to decimal numbers. Binary numbers can be added, subtracted, multiplied together and be divided.



Sexagesimals

Time conversion is a sexagesimals. That is it is related to or based on the number 60 or a system of mensuration to base 60. In division of time into hours, and minutes and seconds we use 60.

When we use 60 as a basis for conversion, we are operating with sexagesimals. Sexagesimals are numbers with base 60, as used in time and minutes and seconds of angle measures.

Identifying Numbers in Equation

Addition $a + b = c$ $4 + 3 = 7$	a is minuend, and b is addend, c is the sum 4 is minuend, 3 is addend, 7 is the sum
Subtraction $a - b = c$ $4 - 3 = 1$	a is minuend, b is subtrahend and c is difference 4 is minuend, 3 is subtrahend, 1 is the difference
Multiplication $a \times b = c$ $4 \times 3 = 21$	a is multiplicand, b is multiplier, c is product 4 is multiplicand, 3 is a multiplier, 21 is the product
Division $a \div b = c$ $6 \div 3 = 2$	a is dividend, b is divisor and c is quotient 6 is dividend, 3 is a divisor, 2 is the quotient

11.1.1.1 Real Numbers

Numbers commonly used in Mathematics are called real numbers. Real numbers and imaginary numbers are subsets of Complex Number System, where the numbers are expressed as $a + bi$. The complex number $a + bi$ can be either real or imaginary.

$2 + \sqrt{2}i$ is a complex number. $2 + \sqrt{2}i = 2 + \sqrt{2} \times \sqrt{-1} = 2 + \sqrt{-2}$

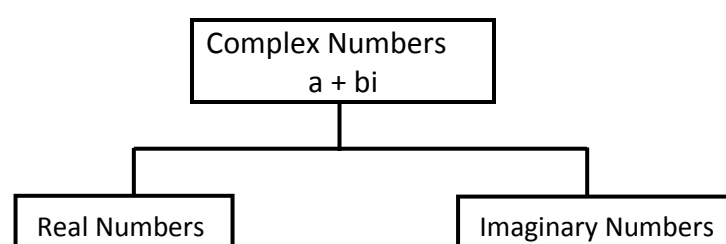
- When $b < 0$ or negative radicand $a + bi$ yields a real number.

$2 + \sqrt{-2}i$ is a real number.

$2 + \sqrt{-2}i = 2 + \sqrt{-2} \times \sqrt{-1} = 2 + \sqrt{2}$ (real, surd)

- When $a = 0$, $b > 0$ or positive radicand $a + bi$ yields an imaginary number.

$\sqrt{-2}$ is imaginary number.



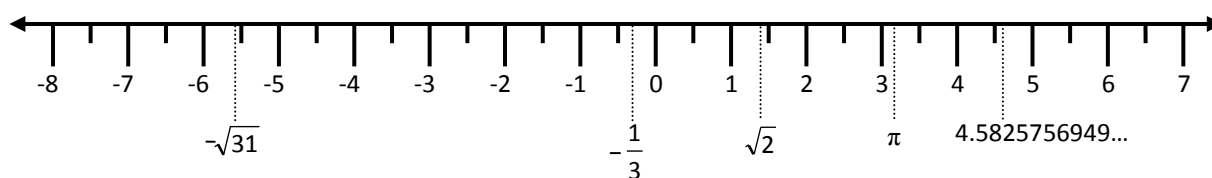


Why are they called real numbers?

They are simply called real numbers because they have concrete values. There is a tangible or physical representation for them. **Real Numbers (R)** are those numbers that are in the form $a + bi$ and can be plotted on a number line.

It means that these numbers 'ib' can be simplified and be expressed in its real number form. Real numbers consist of zero, all negative and all positive numbers including the numbers in between them, which are vulgar fractions, decimal numbers and surds.

The illustration below shows some examples of real numbers and their positions on the number line.



A number between 2 and 3 can be 2.3, 2.41, $2\frac{2}{7}$ or $2 + \sqrt{5}$. The number $2 + \sqrt{5}$ has two values, since $\sqrt{5}$ has positive and negative values.

On the contrary, imaginary numbers cannot be represented on the number line. From the way they are called, they can only be imagined or are unrealistic.

The set of imaginary numbers is used and discussed in higher Mathematics courses. Imaginary numbers contain i , where $i = \sqrt{-1}$ or $i^2 = -1$. And $\sqrt{-1}$ cannot be plotted, but $\sqrt{1} = \pm 1$ can be plotted.

Imaginary numbers are $3i$, $-i$, $-5i$, $6i$ etc. Substituting 'i' with its value for each of the four imaginary numbers we get $3\sqrt{-1}$, $-\sqrt{-1}$, $-5\sqrt{-1}$, and $6\sqrt{-1}$.

Since $i = \sqrt{-1}$, squaring both sides will give $i^2 = -1$. So evaluating $-3i^2$ and $3i^2$ will give real numbers or real solutions, that is, $-3 \times -1 = 3$ and $3 \times -1 = -3$. Where -3 and 3 are integers, rational, and are real numbers.

-3 and 3 are integers	}	Hypothetical Syllogism
Integers are real numbers		If P then Q,
Therefore -3 and 3 are real numbers		If Q then R, So if P then R



In your study of surds you will realize that $\sqrt{1} = -1$ or 1 often stated as $\sqrt{1} = \pm 1$, and $\sqrt{-1}$ has no solution. That is why numbers such as $\sqrt{-1}$, $\sqrt{2}i$, $4i$, $2+$, $\sqrt{-1}$, $\sqrt{3}-\sqrt{-3}$ and $\sqrt{-5}-2$ are called imaginary numbers.

The set of real numbers consists of rational numbers and irrational numbers.

A **rational number** is any number that can be expressed as a ratio (or quotient) of two integers or simply expressed as a fraction. The set of rational numbers includes both integers and fractions. Subset of integers is directed numbers.

An **irrational number** on the contrary cannot be expressed as a fraction which yields to non-terminating and non-repeating decimals.

Transcendental numbers (non - algebraic) such as exponential function (**e**) , gravitational constant (**g**) and pi (**π**) are also irrational numbers.

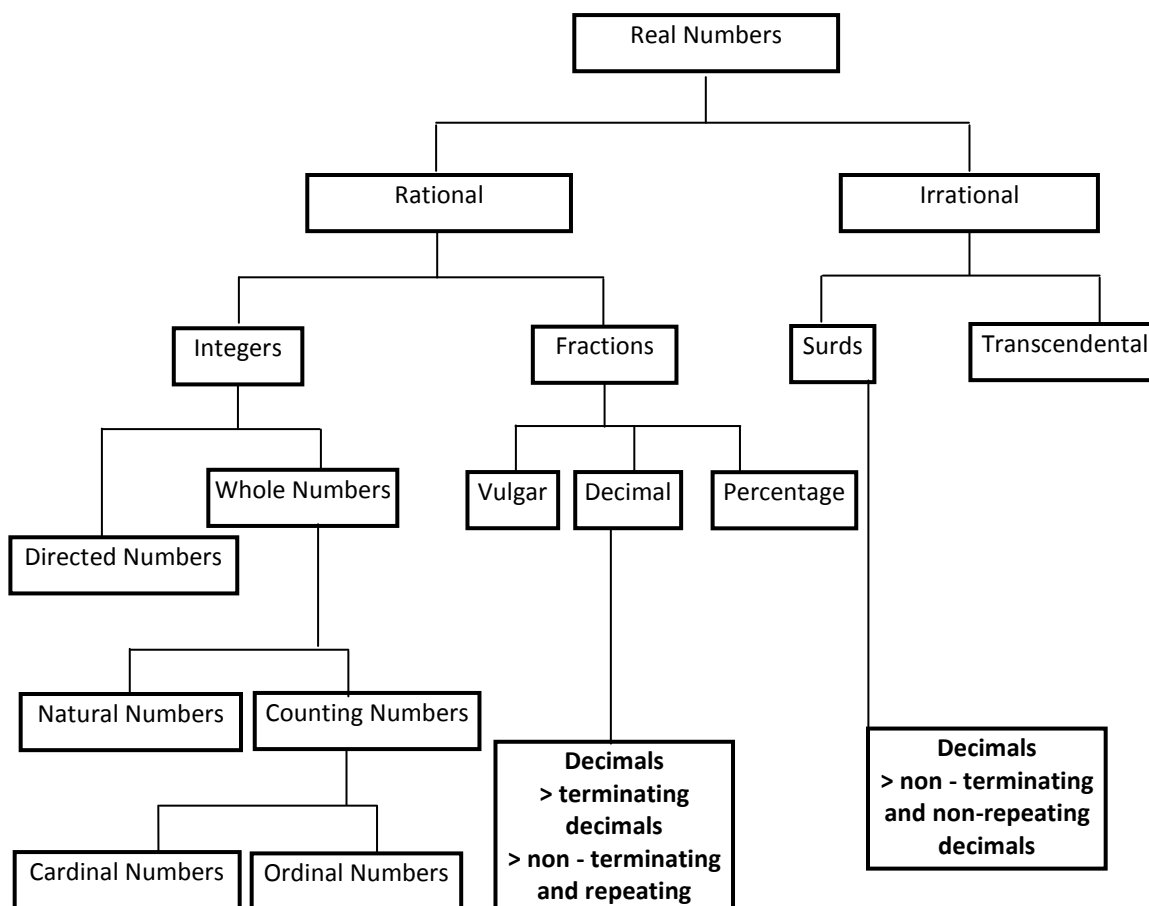
Examples

Rational Numbers	Irrational Numbers
a) The number -3 can be expressed as a fraction $\frac{-3}{1}$ or $-\frac{12}{4}$	a) The value of π (Pi) is 3.1416... which is a non-terminating and a non-repeating decimal
b) The decimal 0.25 can be expressed as a fraction $\frac{25}{100}$ or $\frac{1}{4}$	b) The value of $\sqrt{2} = 1.414213562...$
c) The decimal 0.333333... or sometimes written as $0.\overline{33}$ can be expressed as a fraction $\frac{1}{3}$.	c) The value of $e = 2.718\ 281\ 828\ 459\ 045\ 235...$

The above examples clearly show how rational numbers differ from irrational numbers. The rational (Q) are those that can be expressed in the form $\frac{a}{b}$, $b \neq 0$. Where $a, b \in \mathbb{R}$. When the denominator $b = 0$, the expression is termed as undefined.



The illustration below shows the set of real numbers and how they are related to each other.



The Set of Rational Numbers

The description of each subset and set is given to show that the subset or set to set or subset relation is transitive. [**If a is element of set b, and b is element of set c, then a is element of set c: transitive**].

Rational Numbers ($\mathbb{Q}$) are integers and fractions and all their subsets. They can be expressed in the form a/b , where a and b are integers and $b \neq 0$.

Irrational Numbers include surds and transcendental numbers. They are numbers that cannot be expressed in the form a/b , where $b \neq 0$ and a and b are integers.

Integers ($\mathbb{Z}$) are all negative and positive numbers including zero, $\{-3, -2, -1, 0, 1, 2, 3, 4, 5, \dots\}$ and is a subset of rational numbers.

Fraction is a subset of rational numbers and includes vulgar fractions, decimal fractions and percentage fractions, such as $(\frac{3}{8}, 0.45, 8\%)$ which express part of a whole quantity. A fraction is a number that is not a whole number or combination of a whole number.



Directed Numbers is a subset of integers. They are all positive and negative integers. $\{...-3, -2, -1, 1, 2, 3, 4, 5, ...\}$

Whole Numbers is a subset of integers. It consists of all positive numbers and include zero. $\{0, 1, 2, 3, 4, 5, ...\}$.

Vulgar Fraction is a common fraction and is a subset of fraction. The term vulgar is seldom used. Vulgar or common fractions consist of proper fraction, improper fraction and mixed fraction or mixed number (not placed in the number tree above).

Decimal Fraction is a fraction expressed as a decimal number such as 0.06 and 1.02.

Percentage is a subset of fraction. It is a proportion in one-hundredths, thus expressing part of a whole such as 8% and 27% or any vulgar with denominator of 100.

Decimal Number that is terminating or repeating is a subset of fraction.

Decimal Number that is non-terminating is a subset of surds if it is an algebraic solution, otherwise it is subset of transcendental numbers.

Natural Numbers ($\mathbb{N}$) or counting numbers is a subset of whole numbers. $\{1, 2, 3, 4, 5, ..\}$. The subset of natural numbers among others, are primes and composite numbers.

Cardinal Numbers denote nominal position of something but not in order such as village 4 , PX110, bus 17 etc... $\{1, 2, 3, 4, 5, ..\}$.

Ordinal Numbers denote position of something in a certain order such 3rd village. $\{1^{\text{st}}, 2^{\text{nd}}, 3^{\text{rd}}, 4^{\text{th}}, 5^{\text{th}}, ..\}$.

Transcendental Numbers describes a number or a function that is not algebraic and not the root of algebraic equation. Transcendental numbers are often unplaced on number tree, however are categorized under irrational numbers. Examples of transcendental numbers are e , π and g .

Base System is ... of counting system based on The current study of real numbers is based on the decimal or base 10 counting system which uses numbers from 0 to 9. Base 9 System would use numbers from 0 to 8, base 8 system would use numbers from 0 to 7, etc.

Numerals that we are using in the study of Numbers and Application are Hindu-Arabic. There are also Roman numerals and other numerals. The first ten Roman numerals are I, II, III, IV, V, VI, VII, VIII, IX, X. Often, we use the lower case Roman numerals to number the sequence, such as ii, vi, ix and etc.



Some Mathematical Symbols and Abbreviations

iff means **if and only if** a condition exists (a decimal **€** **Q** **iff** a decimal **€** Fraction)

$a*b = b*a$ (read as 'a operator b equals b operator a' is commutative)

$a*(b*c) = (a*b)*c$ (read as 'a operator, b operator c equals a operator b, operator c' is associative)

$a*(b*c) = ab*ac$ (read as 'a operator, b operator c equals ab operator ac' is distributive)

∞ infinitely large

$\leq$ less than or equal to ($a \leq X$, means all values of **a** be less than or equal to X)

$\geq$ greater than or equal to ($a \geq X$, means all values of **a** be greater than or equal to X)

$a \leq x \leq b$ and $a < x < b$ and $a < x \leq b$ betweenness

$\{x: a \leq x \leq b\}$ x is such that the set ranges from and includes a and b

$\{x: a < x \leq b\}$ x is such that the set ranges from a to b and includes b

$\approx$ is approximately equal to

$\equiv$ same as or congruent to or identity

Σ sum of

Δ triangle (given ΔABC means triangle ABC)

$::$ means

$\bigcirc$ circle

$\rightarrow$ relation such as $A \rightarrow B$, A in relation to B

$\{ \}$ set $\{2,4,6,8,10\}$ a finite set, $\{2,4,6,8,10,\dots\}$ an infinite set

$\in$ element of ($a \in R$ means a is a member of set of real numbers)

$x:$ reads as 'x is such that'

$(*)$ multiply

$(*)$ operation

$\forall$ for all values of (say $\forall a, b$ means for all values of **a** and **b**)

σ mapping or standard deviation

A' image of A

δ delta expresses the change in position or rate.

P Irrational Numbers

Q Rational Number is a set of all fractions and integers.

R Real Number is a complex number with subsets of rational and irrational numbers.

N Natural Numbers are counting numbers.

Z Integer $\{\dots-2, -1, 0, 1, 2, 3, 4, 5,\dots\}$

W Whole numbers

Not all symbols are applied or used in this learning book. Other symbols not used can be used as references.



For any rational numbers, the following properties must apply:

- can be written as a **ratio** of two integers

Example: 1:2

- can be expressed in the form $Q = \frac{a}{b}$, $b \neq 0$; where, a and b are integers.

Example: $\frac{1}{2}$ where $a = 1$ and $b = 2$

- can be positive or negative.

Examples: $-\frac{1}{2}, \frac{1}{-2}, \frac{-1}{2}$

- can have a zero as numerator,

Example: $\frac{0}{2}$ since this number is equal to zero.

When we have a zero as denominator, for example $\frac{1}{0} = \infty$, this number is **undefined** when

$\frac{a}{b} = \infty$, therefore it cannot be considered as a rational number.

Recall that in your previous Mathematics courses, you have discussed already the different sets of numbers. To have a quick revision, let us recall the following definitions:

The most fundamental collection or set of numbers is the set of counting numbers or natural numbers. Definitely, these are the numbers we use for counting. The set of natural numbers consist of $\{1, 2, 3, \dots\}$.

Braces, $\{ \}$, are used to indicate a set of numbers. The three dots after 1, 2, and 3, which are read “and so on”, mean that the elements of the set continuous without ending and is infinite.

The natural numbers, together with the number 0, are called whole numbers. The set of whole numbers consist of $\{0, 1, 2, 3, \dots\}$

Although whole numbers have many uses, they are not adequate for indicating losses or debts. A debt of K60 can be expressed by the negative number -60 (negative 60). When a thermometer reads 5 degrees below zero on a Fahrenheit scale, we say that the temperature is -10 Degree F.

The whole numbers together with the negatives of the counting numbers form the set of integers.



The set of integers consists of $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$. Usually, we see these numbers are represented using the number line.

All the above mentioned sets of numbers are considered as rational for they satisfy the properties of rational numbers and can be proved by use of hypothetical syllogism of the statement

If P then Q, If Q then R, So if P, then R.	→	Ronan is from Bogia Bogia is in Madang Province Joseph is from Madang.
--	---	--

Order of Operations

We work from left to right, if the expression has addition and subtraction only. When there is more than one operation involved, multiply or divide first then add and or subtract.

Example $3 + 4 \div 2$

Solution

$$\begin{aligned} 3 + 4 \div 2 &= 3 + 2 \\ &= 5 \end{aligned}$$

If there is grouping symbol involved, we deal with bracket (), within the bracket we follow order of operations of arithmetic, then multiply or divide and finally add or subtract. Thus the mnemonic BODMAS or BOMDAS can be helpful to memorize the steps.

Example 1 $2(3 + 4) \div 5 - 1$

Solution

$$\begin{aligned} 2(3 + 4) \div 5 - 1 &= 2(7) \div 5 - 1 \\ &= 14 \div 5 - 1 \\ &= 2.8 - 1 \\ &= 1.8 \end{aligned}$$

Example 2 $2 + (3 + 20 \div 5) - 3 \times 2$

Solution

$$\begin{aligned} 2 + (3 + 20 \div 5) - 3 \times 2 &= 2 + (3 + 4) - 3 \times 2 \\ &= 2 + 7 - 3 \times 2 \\ &= 2 + 7 - 6 \\ &= 9 - 6 \\ &= 3 \end{aligned}$$



Properties of Real Numbers

We know that the price of a pencil and the price of a paper are the same as the price of paper plus the price of pencil. But do you know that this example shows to us the commutative property of addition?

The properties of real numbers are usually used to perform the operations of arithmetic and it is better for us to have a thorough understanding of these properties.

The Commutative Properties

When we evaluate $8 + 2$ and $2 + 8$, we will get the same result. This example illustrates the commutative property of addition. Also, for commutative property of multiplication, we can illustrate it just like 12×5 and 5×12 .

Commutative Properties ($m \cdot k = k \cdot m$)

The star or asterisk between m and k and k and m indicate operator in general. That is, it represents plus, minus, multiply or divide, for which operation can hold true is to be proved.

For any real numbers m and k ,

$m + k = k + m$ (commutative property of addition) and

$mk = km$ (commutative property of multiplication)

Example 1 Rewrite each expression and use the commutative property of addition.

a. $3 + ^{-}15$

b. $x^3 + 4$

Solution

a. $3 + ^{-}15 = ^{-}15 + 3$

b. $x^3 + 4 = 4 + x^3$

Example 2 Rewrite each expression and use the commutative property of multiplication.

a. $m \times 7$

b. $(x + 4) \times 8$

c. $2 - ab$

Solution

a. $m \times 7 = 7 \times m = 7m$

b. $(x + 4) \times 8 = 8(x + 4)$

c. $2 - ab = 2 - ba$

**Commutative for subtraction: $m - k \neq k - m$**

The symbol $\neq$ means “is not equal to”.

Illustrative example:

$$9 - 6 = 6 - 9$$

Is this true?

If we are going to solve for this, we get $3 = -3$, which is not true. Therefore, the commutative law does not hold true for subtraction.

Commutative for division: $m \div k \neq k \div m$

Illustrative Example

$$20 \div 5 = 5 \div 20$$

Is this true?

If we are going to solve for this, we get $4 = \frac{1}{4}$ which is not true. Therefore the commutative law does not hold true for division.

The commutative law holds true for addition and multiplication.

The Associative Properties

Let us compute $3 + 5 + 1$. Using the order of operations, we add 3 and 5 to get 8 and then add 8 and 1 to get 9. If we add 5 and 1 first to get 6 and then add 3, we will also get 9. Therefore,

$$(3 + 5) + 1 = 3 + (5 + 1)$$

We get the same answer for either order of addition. This property is the associative property of addition.

The commutative and associative properties of addition are the reason that a burger, a bottle of Coke, and chips cost the same as chips, a hamburger and a bottle of Coke.

We also have an associative property of multiplication. Consider the following two ways of finding the product of 2, 4, and 6.

$$(2 \times 4) \times 6 = 8 \times 6 = 48$$

$$2 \times (4 \times 6) = 2 \times 24 = 48$$

We get the same answer for either arrangement.

**Associative Property** $(m * k) * v = m * (k * v)$

For any real numbers m , k and v ,

$$(m + k) + v = m + (k + v) \text{ and } (mk)v = m(kv)$$

This property means that grouping does not affect the sum or product.

Example 3 Use the commutative and associative properties of multiplication to rewrite each product.

a. $(6x)(x)$ b. $(ab)(3ab)$

Solution

a. $(6x)(x) = 6(x * x)$

$$6x^2 = 6(x^2)$$

$$6x^2 = 6x^2$$

b. The commutative and associative properties of multiplication allow us to rearrange the multiplication in any order. We normally write numbers before the variables, and we usually write variables in alphabetical orders.

$$(ab)(3ab) = 3aabb$$

$$3a^2b^2 = 3a^2b^2$$

Let us inspect this expression below.

$$8 - 5 + 2 - 1 - 2 + 7 - 4$$

Based on the standard **order of operations**, we could evaluate this by computing from left to right. However, using the definition of subtraction, we can re-write this expression as addition of each term.

$$8 + (-5) + 2 + (-1) + (-2) + 7 + (-4)$$

To add these numbers in any order we choose, the commutative and associative properties of addition that will allow us to do so. It is easier to add all the positive numbers; then add all the negative numbers and, then combine these two totals as given below.

$$8 + 2 + 7 + (-5) + (-1) + (-2) + (-4)$$

$$= 17 + (-12)$$

$$= 5$$



Associative for subtraction: $(m - k) - v \neq m - (k - v)$

Example $(12 - 4) - 2 = 12 - (4 - 2)$

$$8 - 2 = 12 - 2$$

$$6 = 10 \text{ is not true.}$$

Therefore associative law does not hold true for the subtraction.

Associative for division: $(m \div k) \div v \neq m \div (k \div v)$

Example $(12 \div 4) \div 2 = 12 \div (4 \div 2)$

$$3 \div 2 = 12 \div 2$$

$$\frac{3}{2} = 6 \text{ is not true.}$$

Therefore associative law does not hold true for division. The associative law holds true for addition and multiplication.

The Distributive Property $a(n + v) = an + av$

For any real numbers a , n and v ,

$$a(n + v) = an + av$$

$$\text{and } a(n - v) = an - av.$$

If four doctors and five nurses pay K3 each for a sandwich, there are two ways to find the total amount spent:

$$3(4+5) = 3 * 9 = 27$$

$$3 * 4 + 3 * 5 = 12 + 15 = 27$$

Since we get K27 on both ways, we can write that

$$3(4+5) = 3 * 4 + 3 * 5$$

We say that the multiplication by 3 is distributed over the addition. This example shows us that the multiplication distributives over addition.

Now, let us take a look at the following expressions involving multiplication and subtraction:

$$8(4 - 2) = 8 * 4 - 8 * 2 \text{ (note the position of the asterisk, which means multiply)}$$

$$32 - 16 = 32 - 16$$

$$16 = 16$$



We can say that both expressions have the same value then, we write

$$8(4 - 2) = 8 * 4 - 8 * 2 \text{ (note the position of the asterisk, which means multiply)}$$

Multiplication by 8 is distributed over each number in the parenthesis.

Let us inspect distribution over multiplication.

$$8(4 \times 2) = 8 \times 4 \times 8 \times 2$$

$$8 \times 8 = 32 \times 16$$

$$64 \neq 512$$

Let us inspect if $a(n \div v) = an \div av$, that is distribution over division. Using the same numbers given in subtraction and multiplication, we write

$$8(4 \div 2) = 8 \times 4 \div 8 \times 2$$

$$8(2) = 32 \div 16$$

$$16 \neq 2$$

This two examples show to us that multiplication does not distribute over multiplication and division.

The distributive law holds true for multiplication over addition and subtraction.

Operations are **commutative** and **associative** over **addition** and **multiplication**; and multiplication **distributes** over **addition and subtraction**.

The Identity Properties

Did you know that numbers 0 and 1 have special properties? Multiplication of a number by 1 does not change the value or the number and addition of 0 does not change the number.

That is the reason why 1 is called the multiplicative identity and 0 is called the additive identity.

Identity Properties ($a*1 = 1*a$ and $a + 0 = 0 + a$)

For any real number m, is such that

$$m * 1 = 1 * m = m \text{ (multiplicative identity) and } m + 0 = 0 + m = m \text{ (additive identity).}$$

Examples

$$8 + 0 = 0 + 8 = 8 \text{ (additive identity)}$$

$$8 * 1 = 1 * 8 = 8 \text{ (multiplicative identity)}$$



Adding “0” to any number will always yield that number; and multiplying any number by “1” will always yield that number.

Other Properties of 0 and 1

If m is a real number not equal to 0 or 1, then the following statements are true.

Identities

$$m + 0 = m$$

$$m \times 1 = m$$

$$m - 0 = m$$

$$m \div 1 = m$$

$$0 \div m = 0$$

$$m \div m = 1$$

$$m \times 0 = 0$$

The Inverse Properties

For every real number, m has an additive inverse; that there exists a real number, $-m$, such that: $m + -m = 0$.

Illustrative Examples

$$7 +^{-} 7 = 0$$

$$\frac{4}{9} +^{-} \frac{4}{9} = 0$$

$$0.06 +^{-} 0.06 = 0$$

For every non-zero real number m , has its additive inverse whose sum equals zero.

For every non zero real number, m also has a multiplicative inverse or reciprocal, written $1/m$, such that their product is equal to 1, that is $m \times \frac{1}{m} = 1$

Illustrative Examples

$$8 \times \frac{1}{8} = 1$$

$$0.53 \times \frac{1}{0.53} = 1$$

$$-3 \times^{-} \frac{1}{3} = 1$$

For any non-zero real number m , has its multiplicative inverse (reciprocal) such that their product equals to 1.



Example Find the multiplicative inverse of

a. 4 b. $-\frac{3}{4}$

Solution

a. The multiplicative inverse of 4 is $\frac{1}{4}$ because $4 \times \frac{1}{4} = 1$

b. The multiplicative inverse of $-\frac{3}{4}$ is $-\frac{4}{3}$. Notice that we only interchanged the positions of the numerator and denominator. $-\frac{3}{4} \times -\frac{4}{3} = \frac{12}{12} = 1$

Showing that the numbers are rational by changing them into a ratio of integers of the form $\frac{a}{b}$ where $b \neq 0$

i. Integers (and Whole numbers) can be expressed as a ratio by using the integer or whole number in the numerator and 1 in the denominator.

Example 1 The number 8 can be written as $\frac{8}{1}$ which is a ratio of the integers 8 and 1.

Example 2 The integer -2 can be written as the ratio $-\frac{2}{1}$.

ii. Proper fractions and improper fractions are also given as a ratio; it is usual to give these vulgar fractions in their simplest form. A proper fraction has its numerator being less than its denominator, otherwise the fraction is improper.

Example 1 The proper fraction $\frac{3}{5}$ is the ratio of 3 and 5.

Example 2 The improper fraction $\frac{14}{6}$ is a ratio but should be written as $\frac{7}{3}$ since it is the simplest form.

iii. Mixed numbers need to be changed to improper fractions. They are fractions containing whole numbers.

Example $1\frac{3}{5}$ can be written as $\frac{8}{5}$; the numerator of the improper fraction is found by multiplying the whole part (1) by the denominator (5) and adding the numerator (3) to give 8, then state the denominator (5).

**iv. Expressing Decimal as a fraction**

In the denominator, place a 1 below the decimal point and add as many zeroes as needed to cover the decimal places (the number of zeroes to be added must be equal to the number of digits on the right of the decimal point). Remove the decimal point from the numerator.

Example 1 0.6 can be written as $\frac{06}{10} = \frac{6}{10} = \frac{3}{5}$

Example 2 0.14 can be written as $\frac{014}{100} = \frac{7}{50}$

Example 3 6.1403 can be written as $\frac{6\ 1403}{10000} = \frac{61\ 403}{10000}$

Example 4 0.0011 can be written as $\frac{00011}{10000} = \frac{11}{10000}$

v. Recurring Decimals

These are decimal numbers that have digits that repeat infinitely after the decimal point in division. The part that repeats is usually shown by placing dots over the first and last digits of the repeating pattern, or sometimes a line over the pattern.

Example 1 $1.\overline{7} = 1.777777\ldots$ 7 is repeated the number.

Example 2 $3.\dot{1}40\dot{6} = 3.140614061406\ldots$ a dot above the beginning and the end of the repeated digits.

Example 3 $0.\overline{1253} = 0.125312531253\ldots$ a bar above the repeated digits.

vi. Expressing Repeated Decimals as a ratio (Vulgar Fraction)

Repeating decimals are rational numbers, so it is possible to express them in the form $\frac{a}{b}$, where $b \neq 0$

- a. In $1.\overline{7} = 1.777777\ldots$ only one number is repeated, that is 7. We write arbitrary equation as $P = 1.777777\ldots$ and $10P = 17.777777\ldots$, thus

$$\begin{array}{r} 10P = 17.777777\ldots \\ P = 1.777777\ldots \quad (\text{subtract}) \\ \hline 9P = 16 \end{array}$$

$$P = \frac{16}{9}$$



- b. In $3.14 = 3.\dot{1}41414...$ we have the numbers 1 and 4 repeating in that order. We write arbitrary equation as $P = 3.141414...$ and $100P = 314.14...$, thus

$$100P = 314.1414...$$

$$\underline{P = 3.1414...} \quad (\text{subtract})$$

$$99P = 311$$

$$P = \frac{311}{99} \text{ simplify where possible or express as mixed number.}$$

$$P = 3\frac{14}{99}$$

- c. In $0.\dot{1}25\dot{3} = 0.125312531253...$ we have the numbers 1, 2, 5 and 3 repeating in that order. We write arbitrary equation as $P = 0.1253 ...$ and $10000P = 1253.1253 ...$, thus

$$10000P = 1253.1253$$

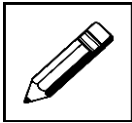
$$\underline{P = 0.1253} \quad (\text{subtract})$$

$$9999P = 1253$$

$$P = \frac{1253}{9999}$$

From the examples b and c, we can observe that the numerator repeats itself if the denominator is 99, 999 or 9999 etc., as long as the number of digits in the numerator is the same as the number of 9's in the denominator.

For example, for the vulgar fractions $\frac{32}{99}$ and $\frac{415}{999}$ will yield $0.323232...$ and 0.415415 respectively.



LEARNING ACTIVITY 11.1.1.1



20 minutes

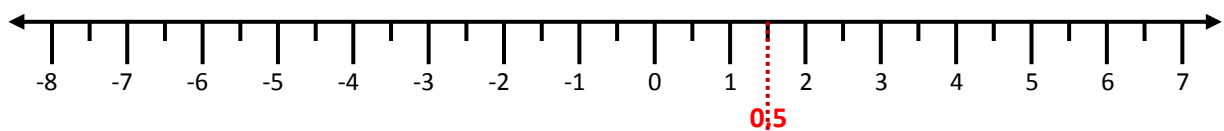
1) Identify if the given number is a rational or irrational number

- a) 0.21 _____
b) 3.1416... _____
c) 1.333... _____
d) $\frac{2}{3}$ _____

2) Identify if the given number is a real or imaginary number

- a) 0.21 _____
b) -45 _____
c) $\sqrt{7}$ _____
d) $\sqrt{-7}$ _____
e) $3i$ _____
f) 13% _____
g) 4^{th} _____
h) $\frac{2}{3}i$ _____
i) $\sqrt{5} - 1$ _____
j) $\sqrt{-1} + 5$ _____
k) π _____
l) e _____
m) 10^{100} (google) _____
n) $10^{10^{100}}$ (googolplex) _____

3) Plot the given Real Numbers on the number line below. The first one is done for you as an example



Example: 0.5

- a) $-2\frac{1}{4}$ d) -7
b) $\sqrt{4}$ e) $5\frac{1}{2}$
c) 2.333... f) π



4) Write on the blank spaces before the letters the properties of real numbers illustrated in the following:

- _____ a) $2 + 3 = 3 + 2$
_____ b) $4(1) = 4$
_____ c) $8 + -8 = 0$
_____ d) $2(4 + 5) = (2 \times 4) + (2 \times 5)$
_____ e) $3 + 0 = 3$
_____ f) $(3 \times 4) \times 6 = 3 \times (4 \times 6)$

5) Write on the blank spaces after the letters the additive and multiplicative inverses of the given real numbers.

- a) 9 _____
b) $2\sqrt[3]{6}$ _____
c) -7 _____
d) $-\sqrt{8}$ _____
e) $\frac{6}{5}$ _____
f) $3\frac{2}{7}$ _____

6) Write equivalents of the fractions to complete the table.

Vulgar	Decimal	Percentage
		45%
	0.62	
$\frac{3}{8}$		
$1\frac{1}{16}$		
	1.05	



11.1.1.2 Factors and Multiples

Factors and multiples are both to do with multiplication. Factors are equal to or less than their multiples. Any multiple will have at least two factors.

Factors are the numbers or variables (pronumerals) we multiply to get a larger number (product).

The following illustration shows 2 and 3 are the factors of 6. In simple terms, we say **factors** are the numbers we can **multiply together** to get another number.

$$\begin{array}{c} 2 \times 3 = 6 \\ \uparrow \quad \uparrow \quad \swarrow \\ \text{Factors} \quad \text{Multiple of both 2 and 3} \end{array}$$

A number can have many factors yet, the factors are finite for any real number.

Example 1 $12 = 12, 6, 4, 3, 2, 1$

Example 2 $26 = 26, 13, 2, 1$

Example 3 $37 = 37, 1$

Factors of 12 are 1, 2, 3, 4, 6 and 12. And because multiplying two negative reals makes a positive, -1, -2, -3, -4, -6 and -12 are also factors of 12. Likewise

$$\begin{aligned} 26 &= 26, 13, 2, 1, -1, -2, -13, -26 \\ 37 &= 37, 1, -1, -37 \end{aligned}$$

But for our purpose, we will use positive factors only in most cases.

$$26 = 26, 13, 2, 1$$

Multiples

Multiples of an integer m are all the integers greater than itself, and are divisible by the integer m . Multiple is the result of multiplying a number by an integer (not a fraction). Unlike factors, multiples are infinite for any real number.

Multiples are numbers that can be divided exactly into by other smaller numbers.

Example List multiples of 3.



Solution

$\{..., -9, -6, -3, 0, 3, 6, 9, 12, ...\}$ (an infinite set)

$\{3, 6, 9, 12, 15, 18, 21, 24, 27, 30\}$ (a finite set; the first 10 positive integer multiples of 3). So we know that 12 is a multiple of 3, and $3 \times 4 = 12$

Multiples of 3

Observe how the multiples of 3 were derived.

-12	-9	-6	-3	0	3	6	9	12
$3(-4)$	$3(-3)$	$3(-2)$	$3(-1)$	$3(0)$	$3(1)$	$3(2)$	$3(3)$	$3(4)$

Example State if the following are multiples of 3?

a) -27

b) 4.5

c) 50

Solution

a) Yes, -27 is a multiple of 3, since $3(-9) = -27$

b) No. Although we can get 4.5 by multiplying 3 by $1\frac{1}{2}$, the result 4.5 cannot be considered as multiple of 3 because it is a product of multiplying 3 with a fraction.

c) No. Because there is no integer to be multiplied by 3 to get 50.

Prime Numbers

Prime numbers or primes are natural numbers divisible by no integers other than unity and itself, such as 2, 3, 5, 7, 11, ..., and -2, -3, -5, ... There are infinitely many prime numbers, but the largest known (August 1989) is $391582 \times 2^{216193} - 1$.

Product of Primes

A natural number can be either composite or prime except 1. The number 1 is neither composite nor prime since it has only one factor.

Composites can be expressed as product of its primes.

Example 1 Express 60 as a product of its prime factors.



Solution

$$\begin{aligned}60 &= 2 \times 30 \\&= 2 \times 2 \times 15 \\&= 2 \times 2 \times 3 \times 5 \\&= 2^2 \times 3 \times 5\end{aligned}$$

Example 2 Express 250 as a product of its primes.

Solution

$$\begin{aligned}250 &= 2 \times 125 \\&= 2 \times 5 \times 25 \\&= 2 \times 5 \times 5 \times 5 \\&= 2 \times 5^3\end{aligned}$$

**LEARNING ACTIVITY 11.1.1.2**

20 minutes

- 1) List the set of factors of the following numbers:

- a) 21
- b) 28
- c) 42
- d) 55
- e) 75

- 2) Write the first ten multiples of the following real numbers.

- a) 25 _____
- b) 14 _____
- c) 31 _____

- 3) Complete the table below by writing the multiples of the following numbers (on the first column) when multiplied by the integers on the first row. The first one is done for you.

x	-3	-2	-1	0	1	2	3
2	-6	-4	-2	0	2	4	6
4							
6							
8							
10							

- 4) List the first five multiples of the following:

- a) 7 _____
- b) 8 _____
- c) 13 _____
- d) 17 _____
- e) 20 _____

- 5) Express as product of their prime factors.

- a) 130 _____
- b) 70 _____
- c) 200 _____



7) Convert decimal system numbers to Binary Number.

- a) 8 _____
- b) 12 _____
- c) 23 _____
- d) 37 _____
- e) 50 _____

8) Express binary numbers as a Decimal system numbers.

- a. 101 _____
- b. 1 011 _____
- c. 11 _____
- d. 10 _____
- e. 110 _____

11.1.1.3 Fractions, Decimals and Percentages

Fraction is a numerical quantity expressing part of a whole quantity. We will use fraction to mean vulgar fraction or common fraction. The following is an example of a fraction.

$$\frac{3}{4}$$

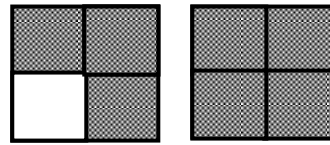
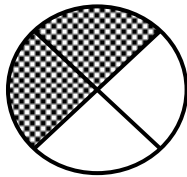
← Numerator

← Denominator

The bottom number (the denominator) says how many parts the whole is divided into.
The top number (the numerator) says how many parts were taken from the whole.



Study the illustration below.



How many parts is the whole circle divided into? How many parts have the two squares been divided into?

The circle shows that 2 out of four parts of the circle has been shaded, which can be re-expressed as $\frac{1}{2}$, in its simplified form.

The squares have been divided into four parts each. Though there are a total of 8 parts, of which 7 are shaded, each square should be considered a complete set with four parts. The square represents a mixed number or improper fraction. Its numeric representation will be written as $\frac{4}{4} + \frac{3}{4} = \frac{7}{4}$ or $1\frac{3}{4}$.

Fractions representing the same quantity are known as **equivalent fractions**.

Forming Equivalent Fractions

Equivalent fractions can be formed by multiplying the numerator and the denominator with the same number or fraction equal to 1. In other words, increase the numerator and the denominator with the same factor.

Example: Determine if $\frac{2}{3}$ is equivalent to $\frac{8}{12}$

There are two ways we can solve for this.

Solution 1

Think of a fraction that is equal to 1 that can be multiplied by $\frac{2}{3}$ to get $\frac{8}{12}$.

Hint: You can use your skills in factors and multiples!

Since 2 is a factor of 8, another factor we need is 4. Yes! Now we try to use $\frac{4}{4}$

$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$ Since the product leads us to $\frac{8}{12}$ then we can say that the two are equivalent fractions.

Solution 2



The second way is a bit easier.

First, equate the two fractions being compared: (Equal sign is used to assume they are equal for the sake of proving).

$$\frac{2}{3} = \frac{8}{12}$$

Second, cross multiply:

$$\frac{2}{3} = \frac{8}{12}$$

$$2 \times 12 = 8 \times 3$$

$$\frac{2}{3} = \frac{8}{12}$$

Take cross products of means and extremes.

Simplify:

$$24 = 24$$

Since $24 = 24$, this is a true equation. We therefore conclude that the two fractions are equivalent.

Example: Determine if $\frac{1}{4}$ is equivalent to $\frac{4}{20}$

Solution

First, equate the two fractions being compared: (Equal sign is used to assume they are equal for the sake of proving).

$$\frac{1}{4} = \frac{4}{20}$$

Second, cross multiply:

$$\frac{1}{4} = \frac{4}{20}$$

$$1 \times 20 = 4 \times 4$$

$$\frac{1}{4} = \frac{4}{20}$$

Take cross products of means and extremes.

Simplify:

$$20 \neq 16$$

Since $20 \neq 16$, we therefore conclude that the two fractions are NOT equivalent.

Fractions can also be expressed in other forms such as decimals and percentages.

Decimals

Decimals is a way of relating to or denoting a system of numbers and arithmetic based on the base ten, tenth parts, and powers of ten.

The numbers we use in everyday life are decimal numbers, which are (0, 1, 2, 3, 4, 5, 6, 7, 8 and 9).

Often "decimal number" is also used to mean a number that contains a decimal point such as 0.201 and 3.25.

Example: 45.6 (forty-five point six) is a decimal number.

$$45.6 = 40 + 5 + \frac{6}{10}$$

You can write decimal fractions with a decimal point (and no denominator), which make it easier to do calculations like addition and multiplication of fractions.



$$\frac{53}{100} = 0.53$$

Example 1 $\frac{3}{7}$ is a common fraction and it can be shown as decimal fraction 0.428571

Example 2 $\frac{43}{100}$ is a common fraction and it can be written as decimal fraction 0.43

Example 3 $\frac{51}{1000}$ is a common fraction and it can be written as decimal fraction 0.051

What did you notice when a fraction whose denominator is a power of 10 is converted to decimals?

Compare the number of decimal places with the number of zeroes in the denominator. Did you see its relationship?

Without using long division, you can easily convert fractions to decimals as long as their denominators are powers of 10. Just make sure that the number of zeroes in the denominator is equal to the number of decimal places you will write in its equivalent decimal number form.

Now study the following.

Examples

Convert the following common fractions to decimal numbers:

a) $\frac{528}{1000}$ b) $\frac{327}{10}$ c) $\frac{3}{50}$

Solution

a) Since the denominator is 1000 (a power of 10), just write 5 and move the decimal point three places to the left. Like this: $\overset{5}{\underbrace{528}_{\text{528}}}$ so the decimal point is now moved before the digit 5 and we say that $\frac{528}{1000}$ can be written in decimal as .528 or 0.528

b) Since the denominator is a power of 10 and there is only 1 zero in it, we write 327 and move the decimal point one place to the left and we have 32.7

c) The denominator 50 is not a power of 10 but we know it is a multiple of 10. So before we do the steps we did in examples a and b, let us first express $\frac{3}{50}$ as a fraction whose denominator is a power of 10. Now we apply our knowledge learnt about equivalent



fractions. Think of a fraction equal to 1 ($\frac{a}{b} = 1$) which will give us a denominator of 100. Did we think of $\frac{2}{2}$? Let us try it. $\frac{3}{50} \times \frac{2}{2} = \frac{6}{100}$. Now we can convert $\frac{6}{100}$ to decimal by moving the decimal point 2 places to the left and we get .06 or 0.06.

Percentages

Percentage is a fraction expressing a quantity out of 100 parts. Percentages can be converted to decimals or common fractions. It is sometimes necessary to convert percentage into common fractions or decimals to ease calculation.

Changing Percentage to Decimals

To change percentage to a decimal number, write the percentage as a fraction then, divide the numerator by the denominator 100.

Example Write 6.5% as a decimal.

Solution

$$\begin{aligned} 6.5\% &= \frac{6.5}{100} \\ &= 0.065 \end{aligned}$$

Changing Percentage to Vulgar Fractions

To change percentage to a vulgar fraction, write the percentage as a fraction then, use the HCF to reduce the fraction to simplify.

Example Write 6.5% as a vulgar fraction.

Solution

$$\begin{aligned} 6.5\% &= \frac{6.5}{100} \\ &= \frac{65}{1000} \\ &= \frac{13}{200} \end{aligned}$$

Changing Vulgar and Decimal Fractions to Percentage

To change a vulgar fraction to percentage, write the vulgar fraction and multiply by 100%. Divide by the denominator and leave the percentage sign.



Example 1 Convert $\frac{5}{8}$ to a percentage.

Solution

$$\begin{aligned}\frac{5}{8} &= \frac{5}{8} \times 100\% \\ &= \frac{500\%}{8} \\ &= 62.5\%\end{aligned}$$

To change a decimal fraction to percentage, write the decimal fraction and multiply by 100%.

Example 1 Convert 1.05 to a percentage.

Solution

$$\begin{aligned}1.05 &= 1.05 \times 100\% \\ &= 105\%\end{aligned}$$

Problems Involving Fractions

Example 1 Mary, Felix and Pat divided K200 as, Mary got a quarter, Felix three-eighths and Pat the rest. How much in kina did Pat get?

Solution

$$\begin{aligned}M &= \frac{1}{4}, F = \frac{1}{3} \quad \text{so } P = 1 - \left(\frac{1}{4} + \frac{1}{3} \right) = \frac{5}{12} \\ \text{Pat} &= \frac{5}{12} \times 200 \\ &= \text{K}83.33\end{aligned}$$

Example 2 Lucy, Schola and Dona piled 800kg of sand for construction work. Lucy carried $\frac{3}{10}$ of the total, Schola $\frac{2}{5}$ and Dona the rest. If a bag holds 10kg, how many bags of sand did Dona carry to the pile of sand?

Solution

$$\begin{aligned}L &= \frac{3}{10}, S = \frac{2}{5} \quad \text{so } D = 1 - \left(\frac{3}{10} + \frac{2}{5} \right) = \frac{3}{10} \\ \text{Dona} &= \frac{3}{10} \times 800 & \text{Bags carried} &= \frac{240\text{kg}}{10\text{kg/bag}} \\ &= 240\text{kg of sand} & &= 24 \text{ bags of sand}\end{aligned}$$



Therefore, Dona carried 24bags of 10kg sand bags to the sand pile.

Example 3 Titus, Cletus, Max and Markus clocked 40 hours, 30 hours, 35 hours and 45 hours in that order in a week. If the company paid them a total of K600, calculate

- (a) The rate per hour,
- (b) Amount each on gets.

Solution

$$\begin{aligned}\text{Total time worked} &= \text{Titus} + \text{Cletus} + \text{Max} + \text{Markus} \\ &= 40 + 30 + 35 + 45 \\ &= 150 \text{ hours}\end{aligned}$$

$$\begin{aligned}\text{a) Rate of Pay} &= \text{Total paid} \div \text{Total time worked} \\ &= K600 \div 150 \text{ h} \\ &= K4/\text{h}\end{aligned}$$

$$\begin{aligned}\text{b) Titus} &= \frac{40}{150} \times 600 & \text{Cletus} &= \frac{30}{150} \times 600 & \text{Max} &= \frac{35}{150} \times 600 & \text{Markus} &= \frac{45}{150} \times 600 \\ &= 4 \times 20 & &= 3 \times 20 & &= 35 \times 20 & &= 45 \times 20 \\ &= K80 & &= K60 & &= K70 & &= K90\end{aligned}$$

Therefore, Titus, Cletus, Max and Markus received K80, K60, K70 and K90 in that order.

Since the rate had been found, rates could have been used to calculate individual wages.

Alternatively, percentage and decimals can be used. However application of both Percentage and decimals, in example 3 may not be convenient, as conversion will yield continuous decimals and not terminating decimals.

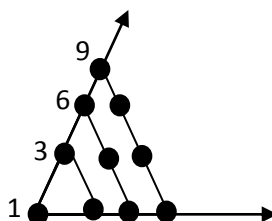
But it is easy to use decimal or percentage if you are using a calculator.

Figurate Numbers

Numbers like Triangular numbers, square numbers and pentagonal numbers are classified as figurate numbers.

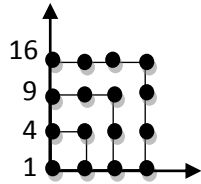
They can be derived by using the rule $2k + \frac{1}{2}nk(k-1) - k^2$ where k is the member of sequence based on n -gons.

Triangular numbers are 1, 3, 6, 9, 12,

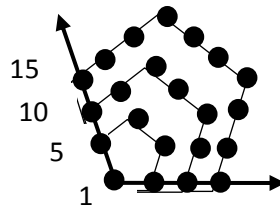




The first fifteen square numbers are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225.



Pentagonal numbers 1, 5, 10, 15 ...



You can build ideas from these examples, how to develop sequences of figurate numbers; based on points nested on regular n -gons from hexagonal numbers and quindecagonal numbers and upwards (which are seldom discussed).

Other Numbers

Fibonacci numbers is the sequence 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ... where each successive term is the sum of the preceding two terms; named after Leonardo Fibonacci (c.1170-1250), a number theorist and algebraist who introduced Arabic number system to Europe.

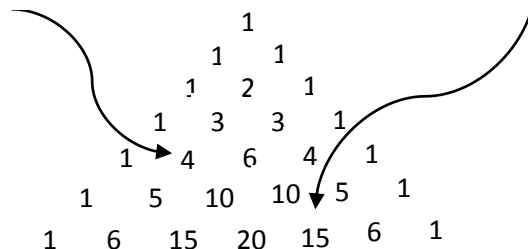
Others such as 2, 2, 4, 6, 10, ... and 4, 4, 8, 12, 20, 32, ... are known as **Lukas Numbers** which were developed by mathematician Lukas, based on Fibonacci number pattern.

Pascals Triangle

Pascals Triangle (discovered by Blaise Pascal, 1623 - 1662) is an array of numbers that enable us to figure out coefficients of polynomial products quickly in expansion of binomials of the form $(x + a)^n$.

4 is a sum of 1 and 3

15 is sum of 10 and 5





Having 1 at the top, 1 at the start and end of each row, the others are found by taking the sum of two numbers above them in the preceding row.

$$\begin{aligned}(x+y)^1 &= x+y \\(x+y)^2 &= x^2+2xy+y^2 \\(x+y)^3 &= x^3+3x^2y+3xy^2+y^3 \\(x+y)^4 &= x^4+4x^3y+6x^2y^2+4xy^3+y^4 \\(x+y)^5 &= x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5 \\(x+y)^6 &= x^6+6x^5y+15x^4y^2+20x^3y^3+15x^2y^4+6xy^5+y^6\end{aligned}$$

$$\begin{aligned}(x+y)^3 &= (x+y)(x+y)^2 \\&= (x+y)(x^2+2xy+y^2) \\&= x^3+2x^2y+xy^2+x^2y+2xy^2+y^3 \\&= x^3+3x^2y+3xy^2+y^3\end{aligned}$$

Likewise, to find $(x+y)^7$ take the product of $(x+y)^6$ and multiply by $(x+y)$.

Interestingly, Pascals Triangle as contains the following patterns in the diagonals:

Counting Numbers {1, 2, 3, 4, 5, 6, ...}

Triangular Numbers {1, 3, 6, 10, 15, ...}

**LEARNING ACTIVITY 11.1.1.3**

20 minutes

1) Encircle ALL fractions equivalent to the given.

a) $\frac{2}{3}$ is equivalent to $\frac{3}{2}, \frac{4}{6}, \frac{6}{12}, \frac{10}{15}, \frac{16}{24}$

b) $\frac{3}{4}$ is equivalent to $\frac{6}{8}, \frac{9}{12}, \frac{12}{15}, \frac{15}{20}, \frac{18}{16}$

2) Convert the following fractions to decimal numbers:

a) $\frac{8}{1000} = \underline{\hspace{2cm}}$

c) $\frac{12}{50} = \underline{\hspace{2cm}}$

e) $\frac{237}{100000} = \underline{\hspace{2cm}}$

b) $\frac{125}{10} = \underline{\hspace{2cm}}$

d) $\frac{9}{20} = \underline{\hspace{2cm}}$

f) $\frac{18}{25} = \underline{\hspace{2cm}}$

3) Convert the following fractions to decimal numbers then to percentage:

Vulgar or Common Fraction	Decimal	Percentage

4) Find the missing terms in the Lukas Number sequence.

0, 3, 3, 6, __, __, 24, __, 63, 102



5) Name the type of the sequence below

a) 36, 49, 64, 81, 100, 121, 144, 169 _____

b) 0, 1, 1, 2, 3, 5, 8, 13, 21, 34 _____

c) 1, 5, 10, 15, 20, 25, 30, 35 _____

d) 5, 5, 10, 15, 25, 40, 65, 105 _____

6) From the 7 rows of array of Pascals Triangle given below, expand the array to 10th row.

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7. Study the 8th row and write the product with correct coefficients of $(x + y)^8$.
 $(x + y)^8 = (x + y) (x + y)^7$

8. Study the 9th row and write the product with correct coefficients of $(x - y)^9$.



9. A tonne of sand is to be supplied by villages according to their population as village A is $\frac{1}{4}t$, village B is $\frac{1}{6}t$, village C is $\frac{3}{8}t$ and the rest village D. What is the total amount in kilograms supplied by villages A and D?

10. A million kina is agreed upon to be the starting capital for a new company set up by three individuals in Felix, Mutchie and Gabby. They pay a kina per share. If Felix pays K450 000, how many shares will he own in the new company?



11.1.1.4 Significant Figures

The term significant figures (sf) refer to the number of important single digits (1 through to 9 inclusive) in the coefficient of an expression in scientific notation. Zero is included only when it lies between two non-zero digits.

The number of significant figures in an expression indicates the confidence or precision with which we state the measure of a quantity. Commonly engineers and scientists in the fields use these expressions. You can also find these expressions in some medicine leaflets and chemical labels.

Significant digits or significant figures are the digits of a number that express a quantity to some specified degree of accuracy, rounding the last figure up if the next would be 5 or greater.

Example 1 Write 3.14159 correct to five significant digits.

Solution

Since $9 > 5$,
 $3.14159 = 3.1416$ (4sf)

Example 2 Write 1.51439 correct to four significant digits.

Solution

Since $3 < 5$,
 $1.51439 = 1.514$ (4sf)

Scientific Notation number is given in the form $A \times 10^N$. A is the coefficient of the number and N is the exponent of base 10.

The exponent N defines the movement of decimal point to the right (exponent is positive) or left (exponent is negative) according to the number in the exponent, when expressing the scientific notation in ordinary number form.

The tables below show several examples of numbers written in standard decimal notation or ordinary form (first column) and in scientific notation (second column). The third column shows the number of significant figures in the corresponding expression in the second column.

**Table 11.1.1.4 (a) Significant Figures in Small Numbers.**

Example Number	Ordinary Form or Decimal Expression	Scientific Notation	Significant Figures
1	0.243	2.43×10^{-3}	3sf
2	0.0000345	3.45×10^{-5}	3sf
3	0.0363637	3.63637×10^{-2}	6sf
4	0.00000009	9×10^{-8}	1sf

From the table above, we notice that every non-zero digit is a significant figure. In the example number 1 there are three significant figures namely 2, 4 and 3 respectively. In the example number 2 there are three significant figures namely 3, 4 and 5.

In the example number 3 there are six significant figures namely 3, 6, 3, 6, 3 and 7. Finally in the example number 4 there is one significant figure which is 9.

Table 11.1.1.4 (b) Significant figures in numbers

Example Number	Decimal Expression Ordinary Form or	Scientific Notation	Significant Figures
1	5	5×10^0	1sf
2	3.48	3.48×10^0	3sf
3	30	3×10^2	1sf
4	63	6.3×10^1	2sf

In example 1, there is only one significant figure which is 5. In example 2, there are three significant figures namely 3, 4, and 8.

In example 3, there is only one significant figure which is 3. In example 4, there are two significant figures namely 6 and 3 respectively.



Table 11.1.1.4 (c) Significant Figures in Large Numbers.

Example Number	Decimal Expression Ordinary Form or	Scientific Notation	Significant Figures
1	6 300	6.3×10^3	2sf
2	72 109	7.2149×10^4	5sf
3	9,876,540,000	9.87654×10^9	6sf
4	8 000 000 000	8×10^9	1sf

In example number 1 there are two significant figures namely 6 and 3 respectively. In example number 2 there are five significant figures namely 7, 2, 1, 0 and 9 respectively.

Zero in between non-zero digits are included (is significant).

In example number 3 there are six significant figures namely 9, 8, 7, 6, 5 and 4 respectively. Finally in example number 4 there is only one significant figure namely which is 8.

Zero as a Significant Figure.

Note that zero or zeroes in between non-zero digits are also counted as significant. Their positions or place values in the decimal number gives value to the decimal number, thus they are considered to be significant.

It is the zeros before the first non-zero digit and the zeros after the last non-zero digit are left out. Non-zero digits are 1, 2, 3, 4, 5, 6, 7, 8 and 9.

Example 1 Count the number of significant figures in the following numerals:

- a) 0.0003210400000
- b) 8.00003×10^{-4}
- c) 2010000000

Solution

- a) In the decimal numeral 0.0003210400000, there are 5 sfs namely 3, 2, 1, 0 and 4.
- b) In the numeral expressed as 8.00003×10^{-4} in scientific notation, there are 5 significant figures namely 8, 0, 0, 0 and 3.
- c) There are 3 significant figures in the numeral 2010000000 namely 2, 0 and 1.



Addition and Subtraction of Scientific Notation Numbers

To find sums of scientific notation numbers

1. Express them in the same power of 10
2. Add or subtract their coefficients, and write the same power of 10
3. Express back to scientific notation
4. Where necessary, write the final answer to significant figures indicated.

Sum of Scientific Notation Numbers or Standard Index Form : $A \times 10^N$

Example Write the sum $2.04 \times 10^4 + 2.04 \times 10^2$ correct to three significant figures.

Solution

$$\begin{aligned} 2.04 \times 10^4 + 2.04 \times 10^2 &= 204 \times 10^2 + 2.04 \times 10^2 \\ &= 206.04 \times 10^2 \\ &= 2.064 \times 10^2 \times 10^2 \\ &= 2.0604 \times 10^4 \\ &= 2.06 \times 10^4 \text{ (3sf)} \end{aligned}$$

Difference of Scientific Notation Numbers or Standard Index Form : $A \times 10^N$

Example Calculate the difference $1.24 \times 10^4 - 2.65 \times 10^3$ correct to two significant figures.

Solution

$$\begin{aligned} 1.24 \times 10^4 - 2.65 \times 10^3 &= 12.4 \times 10^2 - 2.65 \times 10^2 \\ &= (12.4 - 2.65) \times 10^2 \\ &= 9.75 \times 10^2 \\ &= 9.8 \times 10^2 \text{ (2sf)} \end{aligned}$$

Multiplication of Scientific Notation Numbers or Standard Numbers (SIF):

To find products of scientific notation numbers,

1. Multiply the coefficients A
2. Multiply the power of 10
3. Write the product of coefficient A in standard form
4. Simplify the power of 10
5. Write the final answer to significant figures specified.

Example Write the product correct to four significant figures for the area $3.5 \times 10^4 \text{ m} \times 2.64 \times 10^2 \text{ m}$.



Solution

$$\begin{aligned}3.5 \times 10^4 \text{ m} \times 2.64 \times 10^2 \text{ m} &= (3.5 \times 2.64) \times 10^4 \times 10^2 \text{ m}^2 \\&= 9.24 \times 10^4 \times 10^2 \text{ m}^2 \\&= 9.24 \times 10^6 \text{ m}^2 \\&= 9.240 \times 10^6 \text{ m}^2 \text{ (4sf)}\end{aligned}$$

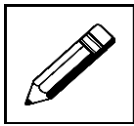
Division of Scientific Notation Numbers or Standard Form: $A \times 10^N$

Example 1.24×10^3 people $\div 3.1 \times 10^5$ ha correct to two significant figures.

Solution

$$\begin{aligned}1.24 \times 10^3 \text{ p} \div 3.1 \times 10^5 \text{ ha} &= 1.24 \times 10^3 \text{ p} \div 3.1 \times 10^5 \text{ ha} \\&= (1.24 \div 3.1) \times 10^3 \text{ p} \times 10^5 \text{ ha} \\&= 0.4 \times 10^{3-5} \text{ p/ha} \\&= 4 \times 10^{-1} \times 10^{-2} \text{ p/ha} \\&= 4.0 \times 10^{-3} \text{ people/ha (2sf)}\end{aligned}$$

The quotients or products of A may not have the decimal point after the first non-zero digit in the operation, so re-write the quotient or product in standard form, then you simplify the powers of 10.

**LEARNING ACTIVITY 11.1.1.4**

20 minutes

1. Complete the table below by identifying the number of significant figures on the second column and naming/listing all the significant figures on the third column.

Given	Number of Significant Digits	List of Digits
a) 0.00002530000		
b) 2.15×10^5		
c) 120 000 000		
d) 12.00045		
e) 15.23000000		
f) 2.05×10^8		
g) 3.4×10^{-3}		
h) 2.000001		
i) 3 000 000 000		
j) 0.0025×10^{-4}		

2. Write the following correct to decimal places and significant figures indicated.

Number	Number of Decimal places	Number of Significant Figures
a) 0.0253	2dp	2sf
b) 2.0652	2dp	3sf
c) 120	1dp	2sf
d) 12.045	3dp	3sf
e) 1.523	3dp	3sf



3. Compute the sums or difference of the following correct to 2 significant figures and express them in ordinary forms. .

a) $2 \times 10^2 + 2 \times 10^{-1} =$

b) $3.4 \times 10^2 + 4.5 \times 10^2 =$

c) $9.32 \times 10^3 - 6.71 \times 10^2 =$

d) $8.56 \times 10^{-1} - 9.7 \times 10^{-2} =$

e) $4.1 \times 10^4 + 2.5 \times 10^3 =$

4. Calculate the products or quotients of the following correct to 3 significant figures and express them in SIF.

a) $2.41 \times 10^2 \times 3.2 \times 10^3 =$

b) $3.2 \times 10^3 \div 4.5 \times 10^2 =$

c) $6.32 \times 10^3 \times 2.7 \times 10^{-2} =$

d) $3.62 \times 10^{-1} \div 7.4 \times 10^{-2} =$

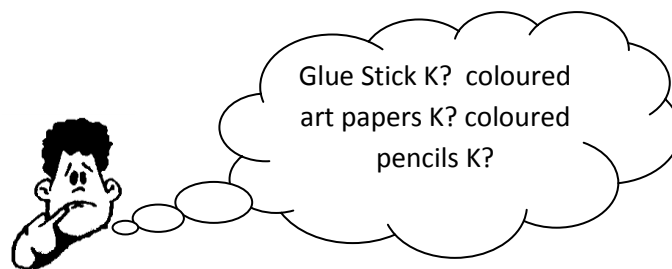
e) $1.4 \times 10^4 \times 5.3 \times 10^3 =$



11.1.1.5 Estimation and Error

In dealing with calculations especially using large numbers, decimals and scientific notations in problem solving, most of the time the exact answer is not always required. This allows one to compute faster and facilitate easier computation especially when a calculator or computing devices are not available.

This skill is known as estimation. To estimate is to make an approximate calculation of something. Example, you are about to go to the shop and you need to buy materials for your project. Your mom asked you how much will you need? Will you be able to give her the exact amount?



You might speedily compute and assume that a glue stick costs around K3, art papers at K10 and colouring materials at K13. An estimate of K26 is needed but you will ask for K30 instead.

This simple situation already involves knowledge on estimation as you made approximate calculations for the amount of glue, art papers and colouring materials. You also made a rounding estimate of the total cost just to make sure that you will not run short if in case the shop assigned higher prices to the materials you will buy.

An estimate is made by rounding the numerals in a calculation to one significant figure. You have learned in your lower secondary mathematics how to round off numbers.

In this lesson, we will focus on the skills when to use specific estimation skills in order to facilitate computation.

Using this idea, we say that $21.4 \approx 21$ if we estimate by rounding off the 21.4 to the nearest whole number. The symbol $\approx$ means “approximately equal to”. And if we use approximation using one significant figure, we say that $21.4 \approx 20$. You will see differences in the actual and the approximated values. This difference is called as the **estimation error**.

Error is the absolute value of the difference between some quantity and an approximation or an estimate of it

When rounding to decimal places and significant figures simultaneously, you will often have different number of digits.



Example 1 Find the estimated sum of K3.58, K10.30, K 29.89 and K 15.75

Solution

Since there is no indicated reference to for estimate (either rounding off using decimal numbers or significant figures), we use both.

Let us use a table to see the comparison of the actual and estimated sums.

Actual Amount	Rounded Amount
K3.58	K4
K10.30	K10
K29.89	K30
K15.75	K20
K59.52	K 64

Degree of error increases when we round off more than once.

The difference between the actual sum and the estimated sum by rounding to the nearest whole is called as the rounding error or **estimation error**. Difference between the actual sum and the estimated sum by rounding to one significant digit is 4.48, which is quite large.

Example 2 Find the estimated product of 28.1 and 845.12 and the estimation error.

Solution

Since it is indicated that we estimate using 1sf, we say $28.1 \approx 30$ and $845.12 \approx 800$. Then, we multiply $30 \times 800 = 24\ 000$.

Therefore, the estimated product when 28.1 and 845.12 is 2400.

$$\begin{aligned}\text{Estimation Error} \quad \text{Exact - Estimate} &= 23\ 747.872 - 24\ 000 \\ &= 252.128\end{aligned}$$

Example 3 Find the estimated quotient when 48.65 is divided by 6.88.

Solution

$$48.65 \div 6.88 \approx 50 \div 7$$



$$\approx 7.11$$

$$\begin{aligned} \text{Estimation Error} \quad \text{Exact - Estimate} &= 7.07 - 7.11 \\ &= 0.04 \end{aligned}$$

Now try the following exercises.



LEARNING ACTIVITY 11.1.1.5



20 minutes

- 1) Complete the table below by estimating the given by rounding off to a whole number (2nd column) and 1 significant figure (3rd column)

Given	Estimated value by rounding off to a whole number	Estimated value by rounding off to 1 significant figure
12.65		
9.42		
15.12		
18.52		
12.86		

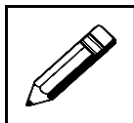
- 2) Find the estimated sum or difference by rounding off to the nearest whole number or 2sf: (The first one is done for you.)

Given	Estimates	Estimated Answer
a) $23.45 + 12.20 + 18.55 =$	$23 + 12 + 19$	$= 54$
b) $58.44 + 28.62 + 11.12 =$		
c) $74.48 - 29.24 =$		
d) $(12.82 - 6.24) + 15.67 =$		

- 3) Find the estimated product or quotient by rounding off to 1sf.

Given	Estimates	Estimated Answer
a) $12.55 \times 89.20 =$		
b) $52.48 \div 47.20 =$		
c) $18.42 \times 64.08 =$		
d) $121.42 \div 4.50 =$		

Congratulations for reaching the end of Topic 11.1.1. Please spend some time to revise what you have learnt in this topic and prepare to answer the following summative task.

**SUMMATIVE TASK 11.1.1**

20 minutes

I. Encircle the letter of the correct answer.

- 1) The following are rational numbers except:
 - A. Fractions
 - B. Integers
 - C. Terminating and repeating decimals
 - D. Non-terminating and non-repeating decimals
- 2) Which of the following real numbers is closest to zero on the real number line?
 - A. $\frac{2}{3}$
 - B. $\sqrt{3}$
 - C. -1.5
 - D. $\frac{\pi}{2}$
- 3) Which of the following shows the Associative Property of Real Numbers?
 - A. $(2 \times 5) = (5 \times 2)$
 - B. $(2 \times 3) \times 5 = 2 \times (3 \times 5)$
 - C. $2(3+5) = (2 \times 3) + (2 \times 5)$
 - D. $(2+3)(5) = (5)(2+3)$
- 4) Which of the following is a multiple of 3?
 - A. 13
 - B. 23
 - C. 33
 - D. 43
- 5) Arrange the following fractions in ascending order $\frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{1}{8}$
 - A. $\frac{3}{5}, \frac{2}{3}, \frac{1}{8}, \frac{1}{2}$
 - B. $\frac{3}{5}, \frac{2}{3}, \frac{1}{2}, \frac{1}{8}$
 - C. $\frac{2}{3}, \frac{1}{8}, \frac{1}{2}, \frac{3}{5}$
 - D. $\frac{1}{8}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}$
- 6) The following fractions are equivalent to $\frac{3}{7}$, except:
 - A. $\frac{6}{14}$
 - B. $\frac{9}{28}$
 - C. $\frac{15}{35}$
 - D. $\frac{18}{42}$



7) Which of the following fractions is equal to the decimal number 0.0012?

- A. $\frac{12}{10}$ B. $\frac{12}{100}$ C. $\frac{12}{1000}$ D. $\frac{10}{10000}$

8) Round off 234.56 to 2 sf.

- A. 231 B. 230 C. 200 D. 100

9) Mike saw the following on his bill after his dinner with his friend.

Rice: K22.50, Chicken: K37.00, Noodles: K 35.50, Drinks: K 25.00

He made a quick estimate by getting the estimated sum using 1 sf. How much is the estimate bill Mike must pay?

- A. K 140 B. K130 C. K120 D. K110

10) Which of the following is the complete set of factors of 30?

- A. 1,2,3,10,15,30 C. 1,2,3,4,5,6,10,15,30
B. 1,2,3,5,6,10,15,30 D. 1,2,3,4,5,6,8, 10,15,30

11) Which of the following options is the binary equivalent of 50?

- A. 11000 B. 110011 C. 110001 D. 110010

12) When 525 is expressed as a product of its primes it yields

- A. $3^2 \times 5 \times 7$ B. $3 \times 5^2 \times 7$ C. $3 \times 5 \times 7^2$ D. $3^3 \times 5 \times 7$

13) Which of the sets is not the set of primes?

- A. 2, 17, 31, 91 B. 7, 41, 51, 53 C. 3, 19, 37, 57 D. 5, 23, 29, 39

14) When we express the binary number 1100100 as a decimal number we get

- A. 1000 B. 500 C. 200 D. 100



II. Complete the table below by identifying the number of significant figures on the second column and naming/listing all the significant figures on the third column.

Given	Number of Significant figures	List
a) 0.21×10^8		
b) 3 000 000		
c) 0.00000001100		
d) 12.4500000000		
e) 1.023×10^8		

III. Complete the table below by estimating the given by rounding off to a whole number (2nd column), 1 significant figure (3rd column) and 2 significant figures (4th column)

Given	Estimated value by rounding off to a whole number	Estimated value by rounding off to 1sf	Estimated value by rounding off to 2sf
325.248			
85.27			
2875.24			
978.005			
29.01			



11.1.2 LAWS OF INDICES

We all know that multiplication is repeated addition. Thus, multiplication makes it easy to deal with same addends. Instead of adding 3 fifteen times, we can simply multiply 3 by 15 and get 45.

The same rule applies if you want to do repeated multiplication. If you want to multiply a number like multiplying 3 by itself five times, you will do $3 \times 3 \times 3 \times 3 \times 3 = 243$. What if you want to multiply it 100 times itself?

Doing $3 \times 3 \times 3 \dots \times 3$ one hundred (100) times may create confusion. But if you say 3^{100} instead, it looks more organized and simplified yet you still do the same process and it will lead you to the same answer.

This topic will lead you to simplified repeated multiplication or what we call as **indices**. You will learn its properties and its practical applications.

11.1.2.1 Basic Concepts

Indices are a useful way of more simply expressing large numbers. They also present us with many useful properties for manipulating them using what are called the **Law of Indices**.

The expression 2^5 is defined as $2^5 = 2 \times 2 \times 2 \times 2 \times 2$. It means 2 is to be multiplied by itself 5 times.

$$\begin{array}{ccc} & \text{base} \rightarrow & 2^5 \quad \leftarrow \text{exponent} \end{array}$$

2^5 is a power, in which the number 2 is called the base and the number 5 is called the index or exponent. The **base** is the number to be multiplied while the **power, index** or **exponent** tells the number of times the base is to be multiplied by itself.

In the given example 2^5 , it only means that 2 is to be multiplied 5 times itself giving the product 32.

Example 1 Simplify the following using index form:

a) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

b) $2 \times 2 \times 2 \times 4 \times 4 \times 4 \times 4 \times 4$

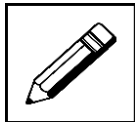
c) $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$



- c) $(2)(2)(2)(2)(2)aaaabbbbbbcc$
e) $\sqrt{5} \times \sqrt{5} \times \sqrt{5} \times \sqrt{5}$

Solutions

- a) Since 3 is multiplied 10 times itself its base is 3 and its exponent is 10 = 3^{10}
b) Since 2 is multiplied 3 times itself and 4 is multiplied 5 times itself = $(2^3)(4^5)$
c) Since $\frac{1}{3}$ is multiplied 5 times itself, we write = $\left(\frac{1}{3}\right)^5$
d) Since 2 is repeatedly multiplied 5 times, a 4 times, b 6 times and cc 2 times, we simplify = $2^5a^4b^6c^2$
e) $\sqrt{5}\sqrt{5}\sqrt{5}\sqrt{5} = (\sqrt{5})^4 = \left(5^{\frac{1}{2}}\right)^4 = 5^2$ Root 5 is a factor 4 times, where root 5 can be expressed as 5 is raised to the power of a half.



LEARNING ACTIVITY 11.1.2.1



20 minutes

- 1) Complete the table below by supplying the missing value using index notation.

Index Form	Base	Exponent
4^8		8
	7	15
$\left(\frac{2}{3}\right)^6$		
	x	5
2^n		

- 2) Simplify the following using index form:

- a) $4 \times 4 \times 4 \times 4 \times 4$ = _____
b) $(-3)(-3)(-3)(-3)(-3)(-3)(-3)(-3)$ = _____
c) $(2)(2)(2)(2)mmmmnnnn$ = _____



3) Expand the following and find their products

	Expanded Form	Product
a) $(\sqrt{3})^6$	= _____	= _____
b) $(-2)^8$	= _____	= _____
c) $(2)^3(\sqrt{3})^2$	= _____	= _____
d) $(\sqrt[3]{8})(\sqrt{9})^3$	= _____	= _____

4) Expand the following and find their products.

	Expanded Form	Product
a) 3^8	= _____	= _____
b) $(-2)^6$	= _____	= _____
c) $(2)^3(3)^2$	= _____	= _____
d) $(\sqrt{3})^3(\sqrt{2})^3$	= _____	= _____

5) Simplify the following.

a) $\left(\frac{2}{5}\right)^4$	= _____	= _____
b) $\left(\frac{\sqrt[4]{81}}{3^2}\right)^{-1}$	= _____	= _____
c) $\left(\frac{\sqrt[5]{3125}}{\sqrt[3]{64}}\right)^2$	= _____	= _____
d) $\left(\frac{\sqrt{6}}{\sqrt{3}}\right)^2 \times \left(\frac{\sqrt{4}}{4}\right)$	= _____	= _____



11.1.2.2 Index Laws of Multiplication

Dealing with powers and indices can be simplified further using the following Laws of Indices:

Law 1. To multiply powers with the same base, add their indices.

$$a^m \times a^n = a^{m+n}$$

The notation above shows that 'a' represents the base while 'm' and 'n' represent the exponents or indices.

Law 1 states that when multiplying powers with the same bases, add or get the sum of the indices.

Example 1 Simplify $5^3 \times 5^2$

Solution

Since both powers have the same base of 5, we write $5^{3+2} = 5^5$

$$\text{Therefore, } 5^3 \times 5^2 = 5^5$$

Example 2 Simplify $2^3 \times 2^4 \times 3^2 \times 3^6$

Solution

Since we can only add the powers of the same bases, we write $2^{3+4} 3^{2+6} = 2^7 3^8$

$$\text{Therefore, } 2^3 \times 2^4 \times 3^2 \times 3^6 = 2^7 3^8$$

Law 2: To raise a power to a power, multiply the powers.

$$(a^m)^n = a^{mn}$$

The second law states that if a power (a^m) is raised to another power (n), then use the same base 'a' raised to the product of m and n.

Example 3 Simplify $(y^3)^4$

Solution

If we will expand the given, it means that y^3 is to be multiplied 4 times itself giving us:

$$y^3 y^3 y^3 y^3$$

If we will apply Law 1 we will have

$$y^{3+3+3+3} = y^{12}$$



Using Law 2 we will only multiply the exponents 3 and 4

$$y^{(3 \times 4)} = y^{12}$$

$$\text{Therefore, } (y^3)^4 = y^{12}$$

Example 4 Simplify $(2^3)^2$

Solution

$$2^{3 \times 2} = 2^6$$

$$\text{Therefore, } (2^3)^2 = 2^6$$

Law 3 To get the power of a product, distribute and multiply the indices

$$(ab)^m = a^m b^m$$

The third law states that if a power consisting of a product **a** times **b** (more than 1 base) is raised to another power (**m**), then both **a** and **b** are raised to **m**.

Example 5 Simplify $(2y)^3$

Solution

The base consists of the product of 2 and y which is $2y$. Since it is raised to the 3rd power, then we raise both 2 and y to the power of 3.

$$(2y)^3 = 2^3 y^3 \\ = 8y^3.$$

Since 2^3 is equal to 8, we further simplify $2^3 y^3$ as

$$\text{Therefore, } (2y)^3 = 8y^3.$$

Example 6 Simplify $(a^2 b^3 c^4)^2$

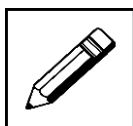
Solution

The example is a combination of Law 3 and Law 2.

Use Law 3 to distribute the power: $a^{2(2)} b^{3(2)} c^{4(2)}$

Law 2 to multiply the distributed powers: $a^4 b^6 c^8$

$$\text{Therefore, } (a^2 b^3 c^4)^2 = a^4 b^6 c^8.$$



LEARNING ACTIVITY 11.1.2.2



20 minutes

1) Simplify the following and indicate the Law of Indices applied.

	Simplified Answer	Law Used
a) $(2x)^3 =$	_____	_____
b) $2^4 2^3 =$	_____	_____
c) $(3^2)^4 =$	_____	_____
	Simplified Answer	Law Used
d) $(-3ab)^2 =$	_____	_____
e) $(2^3)^x =$	_____	_____
f) $x^2 x^3 x^4 =$	_____	_____
g) $(2^3 x^2)^4 =$	_____	_____
h) $x^2 x^3 y^4 y^6 z^8 z^2 =$	_____	_____
i) $(3^b)^4 =$	_____	_____
j) $(3xy)^k =$	_____	_____

2) Simplify the following.

a) $\left(\frac{1}{2x}\right)^{-2} =$ _____

b) $2^{-\frac{1}{2}} \sqrt{2} =$ _____

c) $(x - \sqrt{3})^2 =$ _____

d) $\left(\frac{2^{-2}}{3^{-3}}\right) \left(\frac{\sqrt{32}}{\sqrt{18}}\right)^{-1} =$ _____

e) $\left(\frac{\sqrt{7}}{2\sqrt{3}} x \sqrt{5}\right)^2 =$ _____

f) $x^{\frac{1}{3}} x^{-\frac{2}{3}} x =$ _____

g) $\left(\frac{3}{8}\right)^{-\frac{1}{2}} \left(\frac{8}{3}\right)^{-2} =$ _____

h) $2^{1-2x} \cdot 4^{x-3} =$ _____

i) $10^x \cdot 10^{\frac{1}{2}} =$ _____

j) $\left(\sqrt[k]{xy^2z^3}\right)^k =$ _____



11.1.2.3 Index Laws of Division

Law 4 states that when dividing powers with the same base, simply subtract or get the difference of the indices. This rule applies when the exponent of the dividend (numerator) is greater than the exponent of the divisor (denominator).

Law 4. To divide powers with the same base, subtract their indices.

$$\frac{a^m}{a^n} = a^{m-n} \text{ where } m > n$$

Example 1 Simplify $\frac{y^7}{y^3}$

Solution

We apply Law 4 since the bases in the dividend and divisor are the same and $7 > 3$ (seven is greater than 3). We simply use y as the base and raised to the difference of 7 and 3 that is

$$\frac{y^7}{y^3} = y^{7-3}$$

$$= y^4$$

Therefore, $\frac{y^7}{y^3} = y^4$.

Example 2 Simplify $\frac{a^5b^9}{a^2b^4}$

Solution

Law 4 still applies for this example; however, bear in mind that exponents of the same bases can only be subtracted. So we say:

Simplifying it further:

$$\frac{a^{5-2} b^{9-4}}{a^3 b^5}$$

Therefore, $\frac{a^5b^9}{a^2b^4} = a^3b^5$.

Law 5. To get the power of a quotient, just find the quotient of the powers.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \text{ where } b \neq 0$$

The 5th Law is somehow similar to Law 3 where a power consists of a difference (instead of a product) a divided by b is raised to another power (m), both a and b are raised to m before dividing them or getting the quotient.



Example 3 Simplify $\left(\frac{2}{3}\right)^2$

Solution

Raise both numerator and denominator to 2.

$$\frac{2^2}{3^2} = \frac{4}{9}$$

Therefore, $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$

Example 4 Simplify $\left(\frac{2}{9}\right)^{\frac{1}{2}}$

Solution

$$\begin{aligned}\left(\frac{2}{9}\right)^{\frac{1}{2}} &= \frac{2^{\frac{1}{2}}}{9^{\frac{1}{2}}} \\ &= \frac{\sqrt{2}}{\sqrt{9}} \\ &= \frac{\sqrt{2}}{3}\end{aligned}$$

Since the denominator is a rational number so it is at its simplest form. If the denominator is a surd, rationalize the denominator. When we rationalize the denominator, we make the denominator become a rational number.

Example 5 Simplify the fraction $\frac{2}{\sqrt{8}}$.

Solution

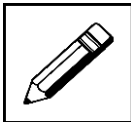
$$\begin{aligned}\frac{2}{\sqrt{8}} &= \frac{2}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} \\ &= \frac{2\sqrt{8}}{\sqrt{64}} \\ &= \frac{2\sqrt{4 \times 2}}{8} \\ &= \frac{2 \times \sqrt{4} \times \sqrt{2}}{8} \\ &= \frac{2 \times 2 \times \sqrt{2}}{8} \\ &= \frac{\sqrt{2}}{2}\end{aligned}$$



Example 6 Simplify $\left(\frac{2^8 \times 3^2}{16}\right)^{\frac{1}{4}}$

Solution

$$\begin{aligned}\left(\frac{2^8 \times 3^2}{16}\right)^{\frac{1}{4}} &= \frac{2^{8 \times \frac{1}{4}} \times 3^{2 \times \frac{1}{4}}}{16^{1 \times \frac{1}{4}}} \\ &= \frac{2^2 \times 3^{\frac{1}{2}}}{16^{\frac{1}{4}}} \\ &= \frac{4 \times \sqrt{3}}{\sqrt[4]{16}} \\ &= \frac{4\sqrt{3}}{2} \\ &= 2\sqrt{3}\end{aligned}$$

**LEARNING ACTIVITY 11.1.2.3**

20 minutes

1) Simplify the following:

a) $\left(\frac{1}{3}\right)^3 =$ _____

b) $\frac{2^5}{2^2} =$ _____

c) $\left(\frac{2x^2y^3}{5z^4}\right)^2 =$ _____

d) $\frac{a^3b^8c^2}{abc} =$ _____

e) $\left(\frac{a^4}{a^2}\right)^3 =$ _____

2) Simplify

a) $\left(\frac{x^2y^4}{z^6}\right)^{\frac{1}{2}} =$

b) $\left(\frac{3\sqrt{2}}{2}\right)^2 =$

c) $\left(\frac{3^3 \times 4^6}{2^3}\right)^{\frac{1}{3}} =$



$$d) \left(\frac{3^3 \times 2^6}{5^3} \right)^{\frac{2}{3}} =$$

3) Simplify the following.

$$f) \left(\frac{2x}{3} \right)^2 = \underline{\hspace{4cm}}$$

$$g) \frac{2^5}{2^2} \times 2 = \underline{\hspace{4cm}}$$

$$h) \left(\frac{2x^6 y^3}{27z^3} \right)^{\frac{1}{3}} = \underline{\hspace{4cm}}$$

$$i) \frac{a^3 b^4 c^5}{a b^2 c^3} = \underline{\hspace{4cm}}$$

$$j) \left(\frac{a^4}{a^2} \right)^{\frac{1}{2}} = \underline{\hspace{4cm}}$$



11.1.2.4 The Rule for Zero as a Power and Negative Power

Law 6: Any number or quantity raised to zero is equal to 1.
 $b^0 = 1$

This Law can actually be derived from Law 4 where powers $m=n$.

For example, we have $\frac{a^2}{a^2}$. If we apply Law 4, we will get a^{2-2} which is actually equal to a^0 .

But how did we make it equal to 1?

Remember the rules in fraction?

When the numerator is exactly the same with the denominator, the fraction is equal to 1. Another basic explanation of this is our simple arithmetic where $1 \div 1 = 1$, $2 \div 2 = 1$, $3 \div 3 = 1$ and so on. So, as long as any number or quantity is divided by itself, the quotient is always 1. This also applies when a number or any quantity is raised to zero.

Example 1 Simplify $(2x^3)^0$

Solution

$$(2x^3)^0 = 2^0 x^0 = 1 \times 1 = 1$$

Example 2 Simplify $(2x^3)^0$

Solution

May mean

$$\begin{aligned}(2x^3)^0 &= \frac{(2x^3)^4}{(2x^3)^4} \\ &= \left(\frac{2x^3}{2x^3} \right)^4 \\ &= 1^4 \\ &= 1\end{aligned}$$

Example 3 Simplify $(2x^3)^0$



Solution

May mean

$$\begin{aligned}(2x^3)^0 &= (2x^3)^5 \cdot (2x^3)^{-5} \\ &= (2x^3)^{5+(-5)} \\ &= (2x^3)^0 \\ &= 2^0 x^0 \\ &= 1 \cdot 1 \\ &= 1\end{aligned}$$

Example 4 Simplify $(2^3 \times \sqrt{6})^0$

Solution

$$\begin{aligned}(2^3 \times \sqrt{6})^0 &= 2^{3 \times 0} \times 6^{\frac{1}{2} \times 0} \\ &= 2^0 \times 6^0 \\ &= 1 \times 1 \\ &= 1\end{aligned}$$

Law 7. Any number or quantity raised to a negative is equal to its positive reciprocal.

$$a^{-m} = \frac{1}{a^m}$$

Example 1 Simplify the following

a) 2^{-2}

b) $\left(\frac{1}{2}\right)^{-3}$

c) $2x^{-2}$

d) $\left(\frac{4^{\frac{1}{2}}}{3^2}\right)^{-2}$

e) $(\sqrt{3})^{-2}$



Solutions

a) Given 2^{-2} , applying Law 7, we get its reciprocal $\frac{1}{2^2} = \frac{1}{4}$

Therefore, $2^{-2} = \frac{1}{4}$

b) $\left(\frac{1}{2}\right)^{-3}$ = we get its reciprocal by dividing 1 by $\left(\frac{1}{2}\right)^3$

$$\frac{1}{\left(\frac{1}{2}\right)^3}$$

Simplify the denominator:

$$\frac{1}{\frac{1}{8}}$$

Divide 1 by $\frac{1}{8}$, we get

$$(1) \times \frac{8}{1} = 8$$

Another way of simplifying it is by getting the reciprocal of the base:

$$\frac{2}{1}$$

and raising it to a positive power 3:

$$\left(\frac{2}{1}\right)^3$$

Apply Law 5:

$$\frac{2^3}{1^3}$$

Simplify:

$$\frac{8}{1} = 8$$

Therefore, $\left(\frac{1}{2}\right)^{-3} = 8$.

c) Given $2x^{-2}$, only x is raised to a negative power so we say: $(2) \cdot \frac{1}{x^2}$

Simplify:

$$\frac{2}{x^2}$$

Therefore, $2x^{-2} = \frac{2}{x^2}$.

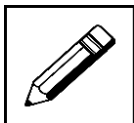


d)

$$\begin{aligned}\left(\frac{4^{\frac{1}{2}}}{3^2}\right)^{-2} &= \frac{4^{-1}}{3^{-4}} \\ &= \frac{3^4}{4^1} \\ &= \frac{81}{4}\end{aligned}$$

e)

$$\begin{aligned}(\sqrt{3})^{-2} &= \frac{1}{(\sqrt{3})^2} \\ &= \frac{1}{\sqrt{3} \times \sqrt{3}} \\ &= \frac{1}{3}\end{aligned}$$

**STUDENT LEARNING ACTIVITY 11.1.2.4**

20 minutes

Simplify the following:

1) $x^0 + 2x^0$ = _____

2) $(2y)^0 + 2y^0$ = _____

3) $\frac{5^0}{3^2}$ = _____

4) $\frac{2x^0}{(2x)^0}$ = _____

5) $(-2x)^0 + (-2)(x)^0 + 4(2^3)^0$ = _____

6) $(-2)^{-2}$ = _____



$$7) 4(x)^{-5} = \underline{\hspace{2cm}}$$

$$8) \left(\frac{2}{7}\right)^{-2} = \underline{\hspace{2cm}}$$

$$9) \left(\frac{y}{x}\right)^{-5} = \underline{\hspace{2cm}}$$

$$10) \left(\frac{x}{2}\right)^{-3} = \underline{\hspace{2cm}}$$

$$11) \left(\frac{8^{\frac{1}{3}}}{2^{-1}}\right)^{-3} = \underline{\hspace{2cm}}$$

$$12) \left(\frac{\sqrt{5}}{5}\right)^{-2} = \underline{\hspace{2cm}}$$

$$13) (-0.5)^{-4} = \underline{\hspace{2cm}}$$

$$14) (-3)^{-6} \times 3^6 = \underline{\hspace{2cm}}$$

$$15) \frac{\left(\sqrt[3]{125} \times \sqrt[4]{81}\right)^{-1}}{\sqrt{225}} = \underline{\hspace{2cm}}$$



11.1.2.5 Indicial Equations

Indicial equations involve equations that have powers as the unknown. To solve indicial equations, the bases on either side of the equal sign must be the same. If they are not, reduce one of the two, so they have the same base. Then, equate the powers and solve for the unknown.

Example 1 If $3^{p+4} = 9^{p-2}$ find the value of p .

Solution

$$\begin{aligned}3^{p+4} &= 9^{p-2} \\3^{p+4} &= 3^{2(p-2)} \\3^{p+4} &= 3^{2p-4}\end{aligned}$$

Since $(p + 4)$ and $(2p - 4)$ are both powers of 3, they must be equal.

$$\begin{aligned}p + 4 &= 2p - 4 \\p - 2p &= -4 \\-p &= -8 \\p &= 8\end{aligned}$$

Example 2 Find the value of x if $(2^{2x})(4^{x+1}) = 64$

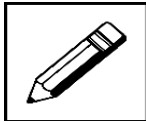
Solution

$$\begin{aligned}(2^{2x})(4^{x+1}) &= 64 \\(2^{2x})(2^{2(x+1)}) &= 8 \times 8 \\(2^{2x})(2^{2x+2}) &= 2^3 \times 2^3 \\(2^{2x+2x+2}) &= 2^{3+3} \\2^{4x+2} &= 2^6\end{aligned}$$

Since $(4x + 2)$ and (6) are both powers of 2, they must be equal.

$$\begin{aligned}4x + 2 &= 6 \\4x &= 4 \\x &= 1\end{aligned}$$

Congratulations for reaching the end of this topic. Please spend some time to revise the lessons you have learnt in this topic and be ready to answer the following summative task.

**STUDENT LEARNING ACTIVITY 11.1.2.5**

20 minutes

Solve for x in the following indicial equations.

1. $8 = 2^x$

2. $\frac{1}{3} = 9^x$

3. $64 = 2^{x+1}$

4. $4(3^x) = 108$

5. $10\,000 = 10^x$



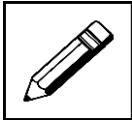
$$6. \quad 3^{x+1} \cdot 9^x = 3^{3x} \cdot 3^{x-2}$$

$$7. \quad 5^{x+2} = 125^x$$

$$8. \quad (7^{x-6})(7^{x+2}) = 2401$$

$$9. \quad 6^x = 6^{x-3} \cdot 6^{x-2}$$

$$10. \quad \frac{1}{100\,000} = 10^x$$

**SUMMATIVE TASK 11.1.2**

20 minutes

Write the correct answer on the spaces provided.

1) It tells the number of times the base is to be multiplied by itself. _____

2) Simplify: $(x)(x)(x)(x)(x)(y)(y)(y)(z)(z)$ _____

3) Write $2^3x^2y^3$ in expanded form. _____

4) Simplify $\left(\frac{x^2y}{z^{-1}}\right)^{-2}$ _____

5) Solve for x in $5^x = 25^{x-2}$ _____

6) Simplify $\left(\frac{3^2}{4^6}\right)^{\frac{1}{2}}$ _____

7) Solve for x in $3^x \cdot 3^2 = 3^5$ _____

8) Simplify $7x^0 + (7x)^0 - 7x$ _____

9) Simplify $3y^2 - 4y^2 + 5y^2$ _____

10) Simplify the following

a) $\left(\frac{1}{2}\right)^{-2} =$ _____

b) $\frac{3^6}{3^2} =$ _____



c) $\left(\frac{x^2 y^3}{z^4}\right)^{-2} =$ _____

d) $\frac{a^8 b^{12} c^7}{a^3 b^2 c} =$ _____

e) $\left(\frac{x^4}{x^4}\right)^3 =$ _____

f) $5x^0 - (2x)^0 =$ _____

g) $-y^0 - 5y^0 =$ _____

h) $\frac{2y^0}{4^2} =$ _____

i) $\frac{(2x)^0}{2x^0} =$ _____

j) $-3(x)^0 + (-2x)^0 + (2^3)^0 =$ _____

k) $(-3)^{-1} =$ _____

l) $12(2x)^{-2} =$ _____

m) $\left(\frac{x}{3}\right)^{-2} =$ _____

n) $\left(\frac{1}{x}\right)^{-8} =$ _____

o) $\left(\frac{2x}{5}\right)^{-1} =$ _____



11.1. 3: SURDS

11.1.3.1 Basic Concepts

Surds are numerical expressions which involve irrational numbers. In some references, they call surds as radicals.

In the previous topics, indices were discussed. In addition to the integer indices you have learned, you should also know that fractions can also be used as indices or exponents.

If you are asked to get the value of $2^{\frac{1}{3}}$, how will you be able to solve it? How will you multiply 2, $\frac{1}{3}$ times itself? This is where knowledge of surds is needed.

In the expression $2^{\frac{1}{3}}$, the base is 2 and the exponent is $\frac{1}{3}$. Now, observe that $2^{\frac{1}{3}}$ can be written or expressed as $\sqrt[3]{2^1}$.

In the expression, note the following equality

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

index radicand exponent of the radicand

Using the expression above:

‘a’ is still called as the base or radicand.

‘n’ is called as the index, taken from the denominator of the fractional exponent.

‘m’ is the exponent of the radicand, taken from the numerator of the fractional exponent.

Note that the calculator can only be used to approximate surds correct to certain number of decimal places. But those are not exact values. The exact values are those that are left as

surds such as $\sqrt{3}$, $2\sqrt{5}$ and $\frac{3\sqrt{3}}{2}$.

So the decimal values below of square roots of natural numbers are approximate values, except for square roots of 1, 4 and 9 which are exact values.



Indices

$$\sqrt{1} = \pm 1.00$$

$$\sqrt{2} = \pm 1.41$$

$$\sqrt{3} = \pm 1.73$$

$$\sqrt{4} = \pm 2.00$$

$$\sqrt{5} = \pm 2.24$$

$$\sqrt{6} = \pm 2.44$$

$$\sqrt{7} = \pm 2.65$$

$$\sqrt{8} = \pm 2.83$$

$$\sqrt{9} = \pm 3.00$$

$$\sqrt{10} = \pm 3.16$$

Example 1 Transform the following fractional exponents to surds:

a) $3^{\frac{1}{2}}$

b) $5^{\frac{3}{4}}$

c) $12^{\frac{2}{5}}$

Solution

Let us use a table to identify the parts and easily transform them as surds.

Indices / Fractional Exponent Form	Base (Radicand)	Denominator of the fractional exponent (index)	Numerator of the fractional exponent (exponent of the radicand)	Surd/Radical Form	Surds read as...
a) $3^{\frac{1}{2}}$	3	2	1	$\sqrt[2]{3^1}$	the square root of 3 raised to 1
b) $5^{\frac{3}{4}}$	5	4	3	$\sqrt[4]{5^3}$	the fourth root of five cubed
c) $12^{\frac{2}{5}}$	12	5	2	$\sqrt[5]{12^2}$	the fifth root of 12 squared

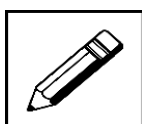
Since you now know how to transform fractional indices to surds, let us do the other way. This time let us transform surds back to fractional exponents. This skill is needed in simplifying surds in the succeeding lessons.



Example 2 Write the following surds and transform them to fractional exponents by completing the table below:

Given	Surd / Radical Form	Radicand (Base)	Index (Denominator of the fractional exponent)	Exponent of the radicand (Numerator of the fractional exponent)	Indices / Fractional Exponent Form
"sixth root of 8 squared"	$\sqrt[6]{8^2}$	8	6	2	$8^{\frac{2}{6}}$
"square root of x raised to the 7 th power"	$\sqrt{x^7}$	x	2	7	$x^{\frac{7}{2}}$
"cube root of 3 squared"	$\sqrt[3]{3^2}$	3	3	2	$3^{\frac{2}{3}}$

Now it is your turn to apply what you have learnt by answering the following learning activity for your practice and mastery.



STUDENT LEARNING ACTIVITY 11.1.3.1



20 minutes

Complete the table below and transform fractional exponents to surds and vice versa.

1) Fractional exponents to surds

Indices / Fractional Exponent Form	Base (Radicand)	Denominator of the fractional exponent (index)	Numerator of the fractional exponent (exponent of the radicand)	Surd/Radical Form	Surds read as...
a) $y^{\frac{1}{3}}$					
b) $2^{\frac{6}{7}}$					
c) $9^{\frac{1}{5}}$					



2) Surds to fractional exponents

Given	Surd/Radical Form	Radicand (Base)	Index (Denominator of the fractional exponent)	Exponent of the radicand (Numerator of the fractional exponent)	Indices / Fractional Exponent Form
"the cube root of x raised to y"					
"the nth root of 3 cubed"					
"the ninth root of 3x raised to 5"					

3) Simplify by expressing the expression $\frac{x^{-1}y^{\frac{1}{2}}}{x^{-2}}$ as a positive surd.

4) Express $\sqrt{a^3b^6c^8}$ in powers.

5) Express $(w^{12}x^6y)^{\frac{1}{3}}$ as a surd.

6) Evaluate $256^{-0.25}$



11.1.3.2 Laws of Surds

In dealing with surds, the following Laws or Rules must be followed to facilitate ease and accuracy in computing.

$$\text{Law 1. } \sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

Law 1 states that the factors of a radicand can be expressed as separate surds.

Example 1 Simplify $\sqrt{48}$

Solution

Since the $\sqrt{48}$ is not a perfect square number, we can get its exact value without using the calculator by just simplifying it. Now, think of a factor of 48 where one is a perfect square number since the surd has an index of 2. (When an index is not indicated, it means it is 2, just like when no exponent of the radicand is indicated means its 1)

Knowing that factors of 48 are 16 and 3, we can re write $\sqrt{48}$ as: $\sqrt{(16 \times 3)}$

Using Law 1: $\sqrt{16} \times \sqrt{3}$

Simplifying the $\sqrt{16}$ and leaving $\sqrt{3}$ as is: $4\sqrt{3}$

Therefore, $\sqrt{48} = 4\sqrt{3}$

Example 2 Simplify $\sqrt[3]{24}$

Solution

Since the $\sqrt[3]{24}$ is not a perfect cube number, we can get its exact value without using the calculator by just simplifying it.

Now, think of a factor of 24 where one is a perfect cube number since the surd has an index of 3.

$$\begin{aligned}\sqrt[3]{24} &= \sqrt[3]{8 \times 3} \\ &= \sqrt[3]{8} \times \sqrt[3]{3} \\ &= 2 \times \sqrt[3]{3} \\ &= 2\sqrt[3]{3}\end{aligned}$$



Therefore, $\sqrt[3]{24} = 2\sqrt[3]{3}$

Law 2. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Law 2 states that the numerator and denominator of a fractional radicand can be expressed as separate surds.

Example 1 Simplify $\sqrt{\frac{3}{4}}$

Solution

Since the radicand of $\sqrt{\frac{3}{4}}$ is not a perfect square number, we can get its exact value without using the calculator by just simplifying it. Since the numerator 3 and itself as its factors, we can no longer find a perfect **square factor/root**, so we will just leave it as a surd. But notice that the denominator 4 is a perfect square number and it is possible to get its square root.

Using Law 2, we write the $\sqrt{\frac{3}{4}}$ as: $\frac{\sqrt{3}}{\sqrt{4}}$

Simplifying the denominator $\sqrt{4}$ and leaving the numerator $\sqrt{3}$ as is: $\frac{\sqrt{3}}{2}$

Therefore, $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

Example 2 Simplify $\sqrt{\frac{27}{12}}$

Solution

$$\begin{aligned}
 \sqrt{\frac{27}{12}} &= \frac{\sqrt{9 \times 3}}{\sqrt{4 \times 3}} \\
 &= \frac{\sqrt{9} \times \sqrt{3}}{\sqrt{4} \times \sqrt{3}} \\
 &= \frac{3 \times \sqrt{3}}{2 \times \sqrt{3}} \\
 &= \frac{3\sqrt{3}}{2\sqrt{3}} \\
 &= \frac{3}{2}
 \end{aligned}$$

Therefore, $\sqrt{\frac{27}{12}} = \frac{3}{2}$



Law 3. $(\sqrt{a})^2 = a$

Law 3 states that if a surd is raised to a power which is the reciprocal of the power, the result is the radicand itself.

Example 1 Simplify $(\sqrt{3})^2$

Solution

To fully understand this law, let us first transform $\sqrt{3}$ as a fractional exponent before we raise it to the second power:

$$(3^{\frac{1}{2}})^2.$$

Now we use Law 2 of Indices (power raised to a power)

$$3^{\frac{2}{2}}$$

Simplifying the exponent, we get:

$$3^1 = 3$$

Therefore, $(\sqrt{3})^2 = 3$.

Example 2 Simplify $(\sqrt[3]{5x})^3$

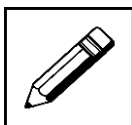
Solution

By just using Law 3, we may simply cancel the exponent and power $(\sqrt[3]{5x})^{\cancel{3}}$ leaving us with $5x$.

Or can be solved algebraically as

$$\begin{aligned}\sqrt[3]{5x} &= \left(5^{\frac{1}{3}}x^{\frac{1}{3}}\right)^3 \\ &= 5^1x^1 \\ &= 5x\end{aligned}$$

Therefore, $(\sqrt[3]{5x})^3 = 5x$.

**STUDENT LEARNING ACTIVITY 11.1.3.2**

20 minutes

1) Simplify the following surds and identify the law used:

	Answer	Law
a) $\sqrt{40}$	= _____	_____
b) $(\sqrt[3]{2x})^3$	= _____	_____
c) $\sqrt[3]{40}$	= _____	_____
d) $\sqrt{\frac{8}{25}}$	= _____	_____
e) $(\sqrt[5]{17x^2})^5$	= _____	_____
f) $\sqrt[3]{\frac{1}{27}}$	= _____	_____
g) $\sqrt{200}$	= _____	_____
h) $\sqrt{\frac{9}{49}}$	= _____	_____
i) $(\sqrt[7]{51})^7$	= _____	_____
j) $\sqrt[3]{270}$	= _____	_____

2) Simplify the following surds and identify the law used:

	Answer	Law
k) $\sqrt{40}$	= _____	_____
l) $(\sqrt[3]{2x})^3$	= _____	_____
m) $\sqrt[3]{40}$	= _____	_____
n) $\sqrt{\frac{8}{25}}$	= _____	_____
o) $(\sqrt[5]{17x^2})^5$	= _____	_____
p) $\sqrt[3]{\frac{1}{27}}$	= _____	_____
q) $\sqrt{200}$	= _____	_____
r) $\sqrt{\frac{9}{49}}$	= _____	_____



s) $(\sqrt[3]{51})^7 =$ _____

t) $\sqrt[3]{270} =$ _____

11.1.3.3 Addition and Subtraction of Surds

Only similar surds can be added or subtracted. This means that when adding or subtracting surds, you must use the same concept as that of adding or subtracting "like" terms in algebra.

The following are similar surds $\sqrt{3}$, $2\sqrt{3}$, and $-4\sqrt{3}$. It is like treating surds as variables like x , $2x$ and $-4x$ in algebra.

Surds are similar if and only if they have the same root and the same radicand

The following are not similar surds: $\sqrt{3}$, $\sqrt[3]{3}$, $\sqrt[4]{3}$ and $\sqrt[5]{3}$, where given are square root, cube root, fourth root and fifth root of 3. Although the surds have similar radicands, the roots are not the same.

How about $\sqrt[3]{3}$, $\sqrt[3]{13}$, $\sqrt[3]{23}$ and $\sqrt[3]{33}$? Are they similar?

No, $\sqrt[3]{3}$, $\sqrt[3]{13}$, $\sqrt[3]{23}$ and $\sqrt[3]{33}$ are not similar surds. Although they have the same roots, their radicands are different.

Since the idea of similar surds is now made known to you, we can now proceed to adding and subtracting the surds.

Example 1 Find the sum of the following:

a) $3\sqrt{6} + 4\sqrt{6}$

b) $-\sqrt{12} + 3\sqrt{3}$

c) $3\sqrt{3} + \sqrt{27}$

Solution

a) Since $3\sqrt{6}$ and $4\sqrt{6}$ are similar surds, add the integers (whole numbers) and write the common radicand: $(3 + 4)\sqrt{6} = 7\sqrt{6}$

b) Since $-\sqrt{12}$ and $3\sqrt{3}$ are not similar surds, let us first write similar form of $-\sqrt{12}$.
Applying Law 1 for surds, we expand and simplify $-\sqrt{12}$ as : $-\sqrt{4} \times \sqrt{3} = -2\sqrt{3}$
We substitute $-\sqrt{12}$ with $-2\sqrt{3}$: $-2\sqrt{3} + 3\sqrt{3}$



We add the integers and write the common radicand: $(-2+3)\sqrt{3}$
Simplify: $1\sqrt{3}$
This can also be written as $\sqrt{3}$. **Therefore, the sum of $-\sqrt{12}$ and $3\sqrt{3}$ is $\sqrt{3}$.**

- c) Since $3\sqrt{3}$ and $\sqrt{27}$ are not similar surds, let us first write similar form of $\sqrt{27}$.
Applying Law 1 for surds, we expand and simplify $\sqrt{27}$ as: $\sqrt{9} \times \sqrt{3} = 3\sqrt{3}$
We substitute $\sqrt{27}$ with $3\sqrt{3}$: $3\sqrt{3} + 3\sqrt{3}$
We add the integers and write the common radicand: $(3+3)\sqrt{3}$
Simplify: $6\sqrt{3}$
Therefore, the sum of $3\sqrt{3}$ and $\sqrt{27}$ is $6\sqrt{3}$.

Example 2 Find the difference of the following pairs of surds:

- a) $4\sqrt{3} - (-2)\sqrt{3}$
b) $2\sqrt{45} - 2\sqrt{5}$

Solution

- a) Since $4\sqrt{3}$ and $-2\sqrt{3}$ are similar surds, get the difference of the integers and write the common radicand:
 $4\sqrt{3} - (-2)\sqrt{3} = (4 - (-2))\sqrt{3}$
 $= (4 + 2)\sqrt{3}$
 $= 6\sqrt{3}$
Therefore, $4\sqrt{3} - (-2)\sqrt{3} = 6\sqrt{3}$.
- b) Since $2\sqrt{45}$ and $2\sqrt{5}$ are not similar surds, let us first write similar form of $2\sqrt{45}$. Applying Law 1 for surds, we simplify $2\sqrt{45}$ as: $(2)\sqrt{9} \times \sqrt{5}$
Get the $\sqrt{9}$: $(2)(3)\sqrt{5} = 6\sqrt{5}$
We substitute $2\sqrt{45}$ with $6\sqrt{5}$: $6\sqrt{5} - 2\sqrt{5}$
We get the difference of the rational coefficients and write the common radicand: $(6-2)\sqrt{5}$
Simplify: $4\sqrt{5}$
Therefore, $2\sqrt{45} - 2\sqrt{5} = 4\sqrt{5}$.

When we say write the radicand, we mean say $\sqrt{5}$, that is 5 is placed under a radical sign and not just 5. The integer 5 is a radicand when it is placed under a radical sign.

Adding and subtracting surds is similar to adding and subtracting like terms in algebra. You can try making the terms similar by simplifying the surds using the laws or rules for surds.

The radicands can be simplified if and only if they are composite numbers. Composite numbers have three or more factors. Prime numbers have only two factors.



A radicand can be simplified if it is not a prime number.

The 25 primes below 100 are

{2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97}. They are natural numbers.

Examples

$$\sqrt{31} = \sqrt{31}$$

$$\sqrt[3]{31} = \sqrt[3]{31}$$

$$\sqrt[4]{31} = \sqrt[4]{31}$$

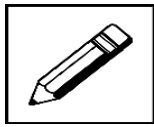
But any prime or prime number raised to a power of a positive integer yields a composite product or number.

Example

$$17^2 = 289 \quad 17 \text{ is prime, } 289 \text{ is composite and } 2 \text{ is the positive integer power.}$$

$$5^3 = 125 \quad 5 \text{ is prime, } 125 \text{ is composite and } 3 \text{ is the positive integer power.}$$

Knowing primes and composites, and your skills in identifying factors and multiples combined with knowledge of like terms will help you ease out work on sums and products of surds.



STUDENT LEARNING ACTIVITY 11.1.3.3



20 minutes

Find the sum or difference of the following surds:

$$1) -11\sqrt{21} - 11\sqrt{21} = \underline{\hspace{2cm}}$$

$$2) -9\sqrt{15} + 10\sqrt{15} = \underline{\hspace{2cm}}$$

$$3) -8\sqrt{5} + 6\sqrt{5} - 9\sqrt{5} = \underline{\hspace{2cm}}$$

$$4) 18\sqrt{7} - 3\sqrt{7} + 7\sqrt{7} = \underline{\hspace{2cm}}$$

$$5) 28\sqrt{3} - 5\sqrt{3} = \underline{\hspace{2cm}}$$

$$6) \sqrt{20} + \sqrt{45} = \underline{\hspace{2cm}}$$

$$7) \sqrt{3} - 5\sqrt{3} = \underline{\hspace{2cm}}$$

$$8) -2\sqrt{20} + 8\sqrt{5} = \underline{\hspace{2cm}}$$

$$9) 12\sqrt{3} - \sqrt{81} = \underline{\hspace{2cm}}$$

$$10) 2\sqrt{20} + 7\sqrt{5} - \sqrt{45} = \underline{\hspace{2cm}}$$

$$11) 2\sqrt{20} - \sqrt{20} + 3\sqrt{20} - 2\sqrt{45} = \underline{\hspace{2cm}}$$

$$12) -3\sqrt{3} - \sqrt{8} - 3\sqrt{3} = \underline{\hspace{2cm}}$$

$$13) \sqrt{27} - \sqrt{48} + \sqrt{147} = \underline{\hspace{2cm}}$$

$$14) \sqrt{23} - \sqrt{41} + \sqrt{7} = \underline{\hspace{2cm}}$$

$$15) \sqrt{225} - \sqrt{169} + \sqrt{49} = \underline{\hspace{2cm}}$$



11.1.3.4 Multiplication of Surds

You can multiply surds by using the Distributive Property or the **FOIL** Method. In both procedures, you can also make use of the Multiplication Property of Surds.

Recall that the product of two radicals is given by $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$, where a and b are real numbers whose n th roots are also real numbers. And your knowledge of $(\sqrt{a})^2 = a$, also applies.

Example 1 Find the product of surds and simplify where possible.

- a) $\sqrt{5} \times \sqrt{10}$
- b) $\sqrt{6} \times \sqrt{6}$
- c) $\sqrt{2} (3 + \sqrt{2})$
- d) $\sqrt{5} (\sqrt{8} - \sqrt{2})$
- e) $(\sqrt{8} - \sqrt{3})(\sqrt{8} - \sqrt{3})$

Solution

- a) Applying the rule in multiplying surds we say: $\sqrt{5} \times \sqrt{10} = \sqrt{50}$
Simplify $\sqrt{50}$: $\sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

Therefore, $\sqrt{5} \times \sqrt{10} = 5\sqrt{2}$

- b) Applying the rule in multiplying surds we say: $\sqrt{6} \times \sqrt{6} = \sqrt{36}$
Since $\sqrt{36}$ is a perfect square: $\sqrt{36} = 6$

Therefore, $\sqrt{6} \times \sqrt{6} = 6$.

- c) Use distributive property of multiplication: $3\sqrt{2} + \sqrt{2} \times \sqrt{2}$
Simplify: $3\sqrt{2} + \sqrt{4}$
But $\sqrt{4}$ can be simplified as 2: $3\sqrt{2} + 2$

Notice that $3\sqrt{2}$ and 2 are not similar surds, therefore we leave them as the final answer and conclude that $\sqrt{2} (3 + \sqrt{2}) = 3\sqrt{2} + 2$. Commutative property of addition applies so it can also be expressed as $2 + 3\sqrt{2}$

- d) Use distributive property of multiplication: $\sqrt{5} \sqrt{8} - \sqrt{5} \sqrt{2}$

Multiply: $\sqrt{40} - \sqrt{10}$
Simplify $\sqrt{40}$ as $\sqrt{4} \times \sqrt{10} = 2\sqrt{10}$ and replace: $2\sqrt{10} - \sqrt{10}$
Get the difference of the integers: $(2-1) \sqrt{10}$



Simplify:

$1\sqrt{10}$

Therefore $\sqrt{5}(\sqrt{8} - \sqrt{2}) = \sqrt{10}$.

- e) For the product of $(\sqrt{8} - \sqrt{3})$ and $(\sqrt{8} - \sqrt{3})$ where the factors are binomials (2 term factors), the product identity is such that the square of binomials yield a three term product: $(a - b)^2 \equiv a^2 - 2ab + b^2$ and $(a + b)^2 \equiv a^2 + 2ab + b^2$. Then simplify to two term expression where possible, when a^2 and b^2 are rational.

$$\text{Multiply: } (\sqrt{8} - \sqrt{3})(\sqrt{8} - \sqrt{3}) = \sqrt{64} - \sqrt{24} - \sqrt{24} + \sqrt{9}$$

$$\text{Simplify } \sqrt{64} - \sqrt{24} - \sqrt{24} + \sqrt{9} \quad \text{as} \quad = 8 - 2\sqrt{24} + 3$$

$$\text{Further simplify} \quad = 8 + 3 - 2\sqrt{4 \times 6}$$

$$= 11 - 2 \bullet 2\sqrt{6}$$

$$= 11 - 4\sqrt{6}$$

Therefore $(\sqrt{8} - \sqrt{3})(\sqrt{8} - \sqrt{3}) = 11 - 4\sqrt{6}$

If the binomial surd factors are contain a common surd factor, the final answer may turn out to be a single rational number such as $(\sqrt{8} - \sqrt{2})(\sqrt{8} - \sqrt{2})$ where these factors can be simplified to $(\sqrt{2})(\sqrt{2}) = 2$. Expanding, than simplifying will give the same result as 2.

The product of surds can be simplified when the product is a multiple of a perfect square.

Say, $\sqrt{5} \times \sqrt{8} = \sqrt{40}$, the radicand of 40 is a multiple of 4. And 4 is a perfect square number, whose square root can be found.

For $\sqrt{5} \times \sqrt{8}$, if we simplify $\sqrt{8}$ to $2\sqrt{2}$ then multiply by $\sqrt{5}$, we will get the same answer as when $\sqrt{40}$ is simplified.

Conjugates

Multiplying surds and simplifying surds at times involve conjugates. The expressions $3 + \sqrt{10}$ and $3 - \sqrt{10}$ are called conjugates of each other where we have a rational and an irrational term.

Conjugates can also have both irrational terms such as the expression $\sqrt{3} + \sqrt{10}$ and its conjugate of $\sqrt{3} - \sqrt{10}$.



The conjugate surds differ only in the operational sign between the two terms. The product of two conjugates is the product of sum and difference of two squares, which is given by the special product formula $(a + b)(a - b) = a^2 - b^2$ or an identity as $(a + b)(a - b) \equiv a^2 - b^2$.

Product of conjugates can be simplified to a single term rational number. This can possibly occur because the product of the first and the last terms are always rational. The middle two irrational (surd) terms happen to be the opposites, so they cancel out.

Algebraic manipulation of conjugates say, a and b are radicands, then

$$\sqrt{a} + \sqrt{b} \text{ has conjugate of } \sqrt{a} - \sqrt{b}$$

$$\text{Thus } (\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - \sqrt{a}\sqrt{b} + \sqrt{a}\sqrt{b} + (\sqrt{b})^2 \quad [\text{four term product}]$$

$$= (\sqrt{a})^2 + (\sqrt{b})^2 \quad [\text{The middle terms are like opposite surd, so they cancel each other or sum up to zero}]$$

$$= a + b \quad [\text{one term, since a and b are rational}]$$

Example 2 Find the conjugate of the given expressions on the first column and find their products.

Expression	Conjugate	Product
$1 - \sqrt{3}$	$1 + \sqrt{3}$	$= (1)^2 - (\sqrt{3})^2$ $= 1 - 3$ $= -2$
$\sqrt{5} + \sqrt{2}$	$\sqrt{5} - \sqrt{2}$	$= (\sqrt{5})^2 - (\sqrt{2})^2$ $= 5 - 2$ $= 3$
$\sqrt{10} - 3$	$\sqrt{10} + 3$	$= (\sqrt{10})^2 - (3)^2$ $= 10 - 9$ $= 1$
$2\sqrt{3} + 3$	$2\sqrt{3} - 3$	$= 4\sqrt{9} - 6\sqrt{3} + 6\sqrt{3} - 9$ $= 4 \bullet 3 - 9$ $= 12 - 9$ $= 3$
$\sqrt{x} + 2$	$\sqrt{x} - 2$	$= (\sqrt{x})^2 - (2)^2$ $= x - 4$

Do you see how simple it is to find the conjugate of a surd?



Comparing the expressions in column 1 with their conjugates in column 2, you will notice that the only difference is their signs. If the expression is a statement of sum (+), then its conjugate is a statement of difference (-) of the two terms or vice versa.

Now, we observe the products in column 3. Note that when the expression and its conjugate were multiplied, they result to a difference of two squares like in the algebraic expression $x^2 - y^2$. That is $(x - y)(x + y) = x^2 - y^2$. Squaring both terms of the surd is easy by just following Law 3 for surds where $(\sqrt{a})^2 = a$.

Example 3 Find the conjugate of the expression $2 - \sqrt{5}$ and multiply the expression by its conjugate.

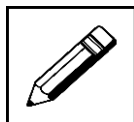
Solution

Since the given is a difference $2 - \sqrt{5}$, its conjugate is a sum $2 + \sqrt{5}$.

Find the product of the two: $(2 - \sqrt{5})(2 + \sqrt{5})$

Following the rules in getting the difference of two squares: $(2)^2 - (\sqrt{5})^2$
Simplify: $4 - 5 = -1$

Therefore, the product of $2 - \sqrt{5}$ and its conjugate $2 + \sqrt{5}$ is -1.



STUDENT LEARNING ACTIVITY 11.1.3.4



20 minutes

1) Find the product of the following and simplify the final answer:

- a) $\sqrt{40} \times \sqrt{10}$ = _____
- b) $\sqrt{15} \times \sqrt{3}$ = _____
- c) $\sqrt{3}(4 + \sqrt{16})$ = _____
- d) $\sqrt{5}(\sqrt{8} + \sqrt{2})$ = _____
- e) $\sqrt{12} \times \sqrt{5}$ = _____
- f) $(-4\sqrt{28})(\sqrt{7})$ = _____
- g) $-3\sqrt{3}(2 + \sqrt{9})$ = _____
- h) $(2 + \sqrt{x})(2 - \sqrt{x})$ = _____
- i) $(\sqrt{11} - 4)(\sqrt{11} + 4)$ = _____
- j) $(-3 - \sqrt{2})(-3 + \sqrt{2})$ = _____



- 2) Complete the table below. In the second column, write the conjugate of the given expression and write their products on the third column.

Expression	Conjugate	Product
a) $2 - \sqrt{5}$		
b) $\sqrt{3} + \sqrt{7}$		
c) $\sqrt{2} + 9$		
d) $-\sqrt{2} + 1$		
e) $-3 + \sqrt{y}$		

- 3) Find the product of the following and simplify the final answer:

a) $\sqrt{2} \times \sqrt{2} \times \sqrt{2}$ = _____

b) $\sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3}$ = _____

c) $\sqrt[3]{4} \times \sqrt[3]{4} \times \sqrt[3]{4}$ = _____

d) $\sqrt[4]{2} \times \sqrt[4]{2} \times \sqrt[4]{2} \times \sqrt[4]{2}$ = _____

e) $\sqrt{5} \times \sqrt{5} \times \sqrt{5} \times \sqrt{5}$ = _____

f) $\sqrt{3} \times \sqrt{12}$ = _____

g) $\sqrt{3} \times \frac{1}{\sqrt{3}} \times \sqrt{15} \times 2\sqrt{3}$ = _____

h) $\frac{\sqrt{y}}{\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{y}} \times \frac{\sqrt{y}}{\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{y}}$ = _____

i) $((2 + \sqrt{11})(\sqrt{11} - 2))$ = _____

j) $(-5 - \sqrt{2})(\sqrt{2} - 5)$ = _____



11.1.3.5 Division of Surds

To simplify a quotient involving surds, you have to rationalize the denominator.

Rationalizing the denominator is a way of removing or eliminating surds in the denominator. An expression is simplified or rationalized when there are no more surds left in the denominator. Usually, this process is used for single term denominator.

The skills you have learnt in the previous lessons will help you in dividing and simplifying surds.

Example 1 Rationalize the denominators of the following.

a) $\sqrt{20} \div \sqrt{100}$

b) $\sqrt{3} \div \sqrt{36}$

Solution

a) The given can be written as:

Simplifying both numerator and denominator gives:

Factor out 2 on both numerator and denominator:

Therefore, $\frac{\sqrt{20}}{\sqrt{100}} = \frac{\sqrt{5}}{5}$.

b) The given can be written as:

Simplifying both numerator and denominator gives:

Since there is no more surd in the denominator and no common factor between the numerator and denominator, we consider $\frac{\sqrt{3}}{6}$ as the quotient of $\frac{\sqrt{3}}{\sqrt{36}}$.

The example 1 shows the direct way of finding the quotient and simplifying them when there is no more surd in the denominator.

The next examples involve dividing surds with single term denominator.

Example 2 Find the quotient of the following:

a) $2 \div \sqrt{3}$

b) $\sqrt{3} \div \sqrt{5}$



Solutions

- a) We all know that $2 \div \sqrt{3}$ can be written as $\frac{2}{\sqrt{3}}$. This expression has a surd in the denominator. To find its simplified quotient, we simply use rationalizing the denominator.

First, multiply the numerator and the denominator by the given denominator

$$(\text{dividend}) \sqrt{3} : \quad \frac{2}{\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}} \right)$$

Get the product of the numerators:

$$2(\sqrt{3}) = 2\sqrt{3}$$

Get the product of the denominators:

$$(\sqrt{3})(\sqrt{3}) = (\sqrt{9}) = 3$$

Since the product of the numerators is $2\sqrt{3}$ and the product of the denominators

is 3, we write our quotient as $\frac{2\sqrt{3}}{3}$.

$$\text{Therefore, } 2 \div \sqrt{3} = \frac{2\sqrt{3}}{3}.$$

Since we already had the detailed explanation of the solution, in example a, let us try a shorter way of rationalizing surds of single term denominator.

- b) Since the given $\sqrt{3} \div \sqrt{5}$ can be written as $\frac{\sqrt{3}}{\sqrt{5}}$, we rationalize the denominator by the following simple steps:

Step 1: Multiply the numerator by the denominator, then simplify: $\sqrt{3}(\sqrt{5}) = \sqrt{15}$

Step 2: Copy the radicand of the denominator.

$$5$$

Since the numerator is $\sqrt{15}$ and the denominator is 5, our quotient is $\frac{\sqrt{15}}{5}$.

$$\text{Therefore, } \frac{\sqrt{3}}{\sqrt{5}} \text{ is rationalized as } \frac{\sqrt{15}}{5}.$$

Both examples (a) and (b) follow the same principle. The method used in example b is just a simplified way of doing so.

Rationalizing Denominators with two terms

Study the given two quotients: $\frac{3}{5+\sqrt{3}}$ $\frac{\text{numerator}}{\text{denominator}}$ $\frac{\sqrt{2}+3}{3-\sqrt{2}}$. The denominators are $5+\sqrt{3}$ and $3-\sqrt{2}$ respectively. The conjugates to be used to rationalize the denominators shall be $5-\sqrt{3}$ and $3+\sqrt{2}$ in that order.

To rationalize a denominator involving two terms, multiply both the numerator and denominator by the conjugate of the denominator.



Example 3 Find the quotient of the following surds.

a) $\frac{\sqrt{3}}{1-\sqrt{5}}$

b) $\frac{4}{2-\sqrt{3}}$

Solution

- a) Step 1: Multiply the numerator and the denominator by the conjugate of the denominator

$$\frac{\sqrt{3}}{1-\sqrt{5}} \times \frac{1+\sqrt{5}}{1+\sqrt{5}}$$

Step 2: Express the numerators and denominators as products.

$$\frac{(\sqrt{3})(1+\sqrt{5})}{(1-\sqrt{5})(1+\sqrt{5})}$$

Step 3: Get the product of the numerators by applying Distributive Law in multiplying surds and get the product of the denominator by multiplying conjugates.

$$\begin{aligned}\frac{(\sqrt{3})(1+\sqrt{5})}{(1-\sqrt{5})(1+\sqrt{5})} &= \frac{\sqrt{3} + \sqrt{15}}{1 + \sqrt{5} - \sqrt{5} - \sqrt{25}} \\ &= \frac{\sqrt{3} - \sqrt{5}}{1 - 5} \\ &= \frac{\sqrt{3} - \sqrt{5}}{-4}\end{aligned}$$

Therefore, $\frac{\sqrt{3}}{1-\sqrt{5}} = \frac{\sqrt{3} + \sqrt{15}}{-4}$ or $\frac{-\sqrt{3} - \sqrt{5}}{4}$

- b) Step 1: Multiply the numerator and the denominator by the conjugate of the denominator.

$$\frac{2}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

Step 2: Express the numerators and denominators as products.

$$\frac{(4)(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$$

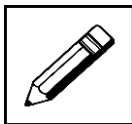
Step 3: Get the product of the numerators by applying Distributive Law in multiplying surds and get the product of the denominator by multiplying conjugates.



$$\begin{aligned}\frac{(4)(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})} &= \frac{8 + 4\sqrt{3}}{4 + 2\sqrt{3} - 2\sqrt{3} - \sqrt{9}} \\ &= \frac{8 + 4\sqrt{3}}{4 - 3} \\ &= \frac{8 + 4\sqrt{3}}{1} \\ &= 8 + 4\sqrt{3}\end{aligned}$$

Therefore, $\frac{2}{2 - \sqrt{3}} = 8 + 4\sqrt{3}$.

Revise the lesson you have just learnt and improve your skills by answering the following learning activity.

**LEARNING ACTIVITY 11.1.3.5**

20 minutes

1) Find the quotient of the following (no need for calculator in this activity):

- a) $\sqrt{50} \div \sqrt{49}$ = _____
- b) $4\sqrt{3} \div \sqrt{16}$ = _____
- c) $12\sqrt{8} \div \sqrt{144}$ = _____
- d) $36 \div \sqrt{81}$ = _____
- e) $-8 \div \sqrt{256}$ = _____

2) Rationalize the following surds with single term denominator:

- a) $\frac{\sqrt{15}}{\sqrt{12}}$ = _____
- b) $\frac{8}{\sqrt{6}}$ = _____
- c) $\frac{\sqrt{4}}{\sqrt{5}}$ = _____

3) Find the quotient of the following surds with two-term denominators: (Use the spaces for your working out.)

a) $\frac{2}{1 - \sqrt{2}}$

Answer: _____



b) $\frac{\sqrt{2}}{2 - \sqrt{6}}$

Answer: _____

c) $\frac{\sqrt{3} - 2}{\sqrt{3} + 2}$

Answer: _____

d) $\frac{\sqrt{3} - \sqrt{7}}{3\sqrt{3} + 1}$

Answer: _____

e) $\frac{\sqrt{3} - \sqrt{5}}{\sqrt{3} + \sqrt{2}}$

Answer: _____

f) $\frac{\sqrt{3} - \sqrt{5}}{\sqrt{3} + \sqrt{5}}$

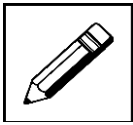
Answer: _____

g) $\frac{-2 + \sqrt{3}}{\sqrt{3} + \sqrt{2}}$

Answer: _____

h) $\frac{\sqrt{3} - \sqrt{5}}{3 - \sqrt{5}}$

Answer: _____

**SUMMATIVE TASK 11.1.3**

20 minutes

Perform the indicated operation(s) involving surds.

1) $-5\sqrt{5} + 12\sqrt{5}$ = _____

2) $\sqrt{40} \times \sqrt{8}$ = _____

3) $\sqrt{5} (3\sqrt{5} - 2\sqrt{5})$ = _____

4) $\frac{\sqrt{16}}{\sqrt{6}}$ = _____

5) $\sqrt{3} (4 + \sqrt{25})$ = _____

6) $-6\sqrt{20} + 2\sqrt{45} + 7\sqrt{5}$ = _____

7) $-4\sqrt{20} + 7\sqrt{5} + 3\sqrt{45}$ = _____

8) $13\sqrt{3} + 18\sqrt{81}$ = _____

9) $\frac{2}{\sqrt{8}}$ = _____



$$10) \quad -\sqrt{2} (\sqrt{3} + \sqrt{27}) \quad = \quad \underline{\hspace{4cm}}$$

$$11) \quad (-8 - \sqrt{2}) (-8 + \sqrt{2}) \quad = \quad \underline{\hspace{4cm}}$$

$$12) \quad 2\sqrt{24} \div \sqrt{144} \quad = \quad \underline{\hspace{4cm}}$$

$$13) \quad \frac{\sqrt{8}}{\sqrt{5}} \quad = \quad \underline{\hspace{4cm}}$$

$$14) \quad (\sqrt{2} + 3) (\sqrt{2} - 3) \quad = \quad \underline{\hspace{4cm}}$$

- 15) Complete the table below. In the second column, write the conjugates of the given expression and write their products on the third column.

Expression	Conjugate	Product
a) $-7 + \sqrt{5}$		
a) $\sqrt{2} + \sqrt{3}$		
b) $\sqrt{2} - 5$		
c) $-\sqrt{6} + 4$		
d) $-2y^2 + \sqrt{y^4}$		



16) Find the quotient when $\sqrt{3}$ is divided by $3 + \sqrt{12}$.

17) Simplify: $\frac{\sqrt{32}}{4 - \sqrt{3}}$

18) $(\sqrt{5} - 6)(3 + \sqrt{3}) =$

19) $(\sqrt{5} - \sqrt{3})(\sqrt{2} + \sqrt{3}) =$

20) Add the conjugate of $3 + \sqrt{5}$ to $\sqrt{180}$. Simplify the final answer.



11.1. 4: UNITS OF MEASUREMENTS

Weights and measures were among the earliest tools invented by man. Primitive societies needed rudimentary measures for many tasks: constructing dwellings of an appropriate size and shape, fashioning clothing and bartering food or raw materials.

Man understandably turned first to parts of his body and his natural surroundings for measuring instruments. Early Babylonian and Egyptian records and the Bible indicate that length was first measured with the forearm, hand, or finger and that time was measured by the periods of the sun, moon, and other heavenly bodies.

When it was necessary to compare the capacities of containers such as gourds or clay or metal vessels, they were filled with plant seeds that were then counted to measure the volumes. With the development of scales as a means for weighing, seeds and stones served as standards.



In the primitive years, people used their body parts to measure objects; however, this system may not be fair because people have different body dimensions.



Ancient container used to measure volume.



Plant seeds used to fill in ancient containers which were counted to measure volume.

For instance, the "carat," still being used as a mass unit for gems, is derived from the carob seed. As societies evolved, measurements became more complex. The invention of numbering systems and the science of mathematics made it possible to create whole systems of measurement units suited to trade and commerce, land division, taxation, and scientific research. For these more sophisticated uses, it was necessary not only to weigh and measure more complex things but also necessary to do it accurately time after time and in different places.

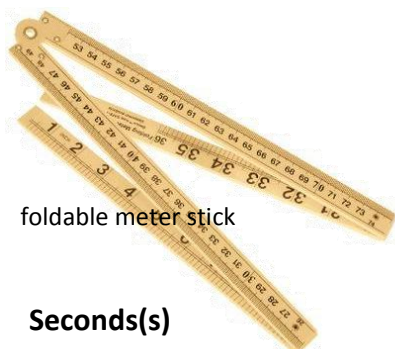
However, with limited international exchange of goods and communication of ideas, it is not surprising that different systems for the same purpose developed and became established in different parts of the world - even in different parts of the same country.

At present, there are two commonly used and accepted standards or systems in measurement: the Imperial System and the Metric System.



The Metric system is also known as the International System of units or SI. It is widely used all over the world because it uses bases which are multiples of 10. This gives an easier way of converting from one unit to another among the units of length, mass and volume.

The following are some of the base units of SI, with some commonly used measuring tools:

Metre (m)

foldable meter stick

Kilogram (kg)

weighing scale

Litre (L)

measuring cup

Seconds(s)

analogue clock

Degree Celcius

digital thermometer

11.1.4.1 Measurement

Measurement is a process of comparing the dimensions of an object using a standard way of quantifying things. When we say standard, we mean something that is agreed upon and commonly used by many.

The Imperial System of units was defined in 1824 and was used by many countries in the British Empire. But before the 20th century ended, some of these countries had converted to Metric System.

At present, there are two commonly used and accepted standards or systems in measurement: the Imperial System and the Metric System or known as the International System of Units (SI).



The International System of Units (SI) defines the following seven units of measure as a basic set from which all other SI units are derived:

Measurement	Units of Measure
length	Metre
mass	Kilogram
time	Second
electric current	Ampere
temperature	Kelvin
luminous intensity	Candela
amount of substance	Mole

Derived units from these fundamental units given in the above table are Newton (N), Joule (J), Watt (W) and Pascal (Pa).

Multiples and fractions of the basic units are defined in multiples of 1000, and are denoted by following prefixes (and symbols).

10^3	kilo-	(k)	10^{-3}	milli-	(m)
10^6	mega-	(M)	10^{-6}	micro-	(μ)
10^9	giga-	(G)	10^{-9}	nano-	(n)
10^{12}	tera-	(T)	10^{-12}	pico-	(p)
10^{15}	peta-	(P)	10^{-15}	femto-	(f)
10^{18}	exa-	(E)	10^{-18}	atto-	(a)

In addition the following customary prefixes are used:

10	deka-	(da)	10^{-1}	deci-	(d)
10^2	hecto-	(h)	10^{-2}	centi-	(c)

Another unit used to measure capacity is litre. However, this is not yet formally part of SI but it is accepted for use with the SI. While litre is the basic unit for liquid volume, cubic centimetre is the basic unit for solid volume.

In the metric system, each quantity measured has a basic unit of which other units were based upon using prefixes.



To easily remember these prefixes, we may use the mnemonics **K**arl **H**as **D**eveloped **M**y **D**ecimal **C**ravings for **M**etrics. This mnemonics will help you easily remember the first letters of the prefixes used in the metric system.

Kilo (1000)	Hector (100)	Deka (10)	metric base unit	Deci (1/10)	Centi (1/100)	Milli (1/1000)
kilometre	hectometre	dekametre	Metre	decimetre	centimetre	millimetre
kilogram	hectogram	dekagram	Gram	decigram	centigram	milligram
Kilolitre	hectolitre	dekalitre	Litre	decilitre	centilitre	millilitre

The common among prefixes used are: kilo, centi and milli. However, knowledge on the other prefixes will be beneficial because they are used in some specific areas such as in the fields of medicine, engineering, architecture, physics, chemistry and the like.

In computing, an external drive can store up to one terabytes of information.

The table below shows some of the commonly used units in both Metric and Imperial systems.

Quantity it measures	Metric	Imperial
Distance	Metre(m) kilometre (km), centimetre(cm), millimetre(mm)	inch (in) foot(ft) yard(yd) mile(mi)
Weight	gram (g) kilogram (kg) milligram (mg)	Ounce(oz) pound(lbs) ton (t)
Capacity	litre (L) millilitre (mL)	Gallon(gal) Quart(qt) Pint(pt)
Pressure	Kilopascals (kPa)	Pounds per square inch (psi)

**STUDENT LEARNING ACTIVITY 11.1.4.1**

20 minutes

- 1) What is the importance of having and following a common standard of measurement?

- 2) List some measuring tools you use at home and identify if they are using the SI or imperial units.

- 3) Describe an analogue instrument.

- 4) What system of units do you commonly use in your school?

- 5) Write one teragram in figures. How many kilograms would that be equal to?

- 6) When can you use nanometres to measure?

- 7) Are these prefixes relevant in time measurement?

- 8) What do we mean by saying derived units?

- 9) Write the symbols for kelvin, candela and mole.

- 10) When can you use nanometres to measure?



11.1.4.2 Conversion of Metric and Imperial Units

Sometimes you will need to convert or change from one system to the other especially when you travel abroad because one country may use both the imperial and SI units. In addition, the labels on the goods you usually find in the shops use either of the two units.

It is also important to learn how to convert from one unit to another because your job in the near future may require you to do so. To do this, you need a conversion key that lists equivalent measurements.

If you need to convert units between systems when working on a job, you are usually provided with the equivalents, but you will save yourself time if you know the most common equivalents. So it will be very helpful if you try your best to learn these by heart.

Before we start converting between the two systems, we will briefly review how to convert within either of the systems.

Converting Within the Imperial System

The table below shows the equivalents used when converting within the imperial system.

IMPERIAL EQUIVALENTS	
1 foot (ft)	12 inches (in)
1 yard (yd)	3 feet (ft)
1 mile (mi)	1760 yard (yd)
1 pound (lb)	16 ounces (oz)
1 quart (qt)	2 pint(pt)
1 gallon (gal)	4 quart (qt)

In this table, the larger units are on the left. Tables might be set up with the larger units on the right, so the first line would read 1 foot = 12 inches.

However, one unit, usually the larger unit, has a 1 in front of it (like 1 ft, 1 yd, 1 mi). The other unit has a number different than 1 in front of it (like 12in, 3ft, 1760 yd).

This number is the **conversion factor**. To convert an imperial quantity in one unit to another unit, follow these two rules.



Rule 1:

To convert from a larger unit such as feet to a smaller unit such as inches, multiply the amount of the larger unit by the conversion factor.

Example 1 Convert 5 feet to inches.

Solution

The chart tells you that 1 foot = 12 inches. The conversion factor is 12. To convert 5 feet to inches (larger unit to smaller unit), multiply by the conversion factor 12.

$$\begin{aligned}1 \text{ ft} &= 12 \text{ in} \\5 \text{ ft} &= 5 \text{ ft} \times 12 \text{ in/ft} \\&= 60 \text{ in}\end{aligned}$$

Example 2 Convert 6 miles to yards.

Solution

The conversion key tells you that 1 mile = 1760 yards. The conversion factor is 1760. To convert 6 miles to yards (larger unit to smaller unit), multiply by the conversion factor 1760.

$$\begin{aligned}1 \text{ mile} &= 1760 \text{ yd} \\6 \text{ mi} &= 6 \text{ mi} \times 1760 \text{ yd/mi} \\&= 10\,560 \text{ yd}\end{aligned}$$

Rule 2:

To convert from a smaller unit such as quarts to a larger unit such as gallons, divide the amount of the smaller unit by the conversion factor.

Example 1 Convert 10 quarts to gallons.

Solution

From the chart, 1 gallon = 4 quarts. The conversion factor is 4. Since you are going from a smaller unit to a larger one, you divide by 4.

$$\begin{aligned}1 \text{ gal} &= 4 \text{ qt} \\10 \text{ qt} &= 10 \text{ qt} \div 4 \text{ qt/gal} \\&= 2.5 \text{ gal}\end{aligned}$$



Example 2 Convert 96 ounces to pounds.

Solution

From the conversion key table, $16\text{oz} = 1\text{lb}$. The conversion factor is 16. Since you are going from a smaller unit to a larger one, you divide by 16.

$$\begin{aligned}1\text{ lb} &= 16\text{oz} \\96\text{oz} &= 96\text{oz} \div 16\text{oz/lb} \\&= 6\text{lb}\end{aligned}$$

Converting within the Metric System

To convert within the metric system, you multiply or divide by 10, or a power of 10 such as 100 or 1000. To do this, you move the decimal point to the left or right the required number of places, using zeros as place holders when necessary. (You are actually multiplying or dividing by a power of ten when you move a decimal point.)

The table below lists the most commonly used prefixes with the basic units for weight (gram), length (metre), and capacity (litre).

Metric Equivalents

Prefix	Value	Basic Unit
Kilo	1000	gram metre litre
Centi	$1/100 = 0.01$	gram metre litre
Milli	$1/1000 = 0.001$	gram metre litre

To convert to or from the basic units of gram, meter or litre, look at the prefix in front of the non-basic unit to tell how many places to move the decimal point.

1. The prefix kilo means 1000 times the basic unit.

To change from a unit with kilo as its prefix to a basic unit, or to convert from a basic unit to a unit with kilo as its prefix, move the decimal point 3 places.

2. The prefix centi means .01 times the basic unit.

To change from centi to a basic unit, or to convert from a basic unit to a unit with centi as its prefix, move the decimal point 2 places.



3. The prefix milli means 0.001 times the basic unit.

To change from a unit with milli as its prefix to a basic unit, or to convert from a basic unit to a unit with milli as its prefix, move the decimal point 3 places.

What direction do you move the decimal point to complete the conversion?

Use the following rules:

1. To convert from a smaller unit to a larger one, move the decimal point to the left.
2. To convert from a larger unit to a smaller one, move the decimal to the right.
3. The number of places to move the decimal point in the original amount when converting to or from a basic unit depends on the prefix of the other unit.
 - With the prefix kilo, move the decimal three places.
 - With the prefix milli, move the decimal three places.
 - With the prefix centi-, move the decimal two places.

**To convert from a large unit to a smaller unit, multiply.
To convert from a smaller unit to a larger unit, divide.**

Example 1 Convert 650 millimetres to metres.

Solution

Converting from a smaller unit to a larger one, the decimal point moves to the left. The prefix milli- indicates that the decimal point moves three places when going to or from a basic unit. Move the decimal point three places to the left. Change the unit to meters.

$$650 \text{ mm} = 0.650 \text{ m}$$

Example 2 Convert 8450 metres to kilometres.

Solution

$$1000\text{m} = 1\text{km}$$

$$\begin{aligned} 8450\text{m} &= 8450\text{m} \div 1000\text{m/km} \\ &= 8450\text{m} \times \text{km}/1000\text{m} \\ &= 8.45\text{km} \end{aligned}$$



Example 3 Convert 3.82kilograms into grams.

Solution

$$1000\text{g} = 1\text{kg}$$

$$\begin{aligned} 3.82\text{kg} &= 3.82\text{kg} \times 1000\text{g/kg} \\ &= 3820\text{g} \end{aligned}$$

Example 4 Convert 1.32kilolitres to millilitres.

Solution

$$1\,000\text{L} = 1\text{kL}$$

$$\begin{aligned} 1.32\text{kL} &= 1.32\text{kL} \times 1\,000\text{L/kL} \\ &= 1\,320\text{L} \end{aligned}$$

$$1\,000\text{mL} = 1\text{L}$$

$$\begin{aligned} 1\,320\text{L} &= 1\,320\text{L} \times 1000\text{mL/L} \\ &= 1\,320\,000\text{mL} \end{aligned}$$

CONVERTING BETWEEN METRIC AND IMPERIAL UNITS

Sometimes you need to convert, or change, from one system to the other. To do this, you use a chart, like the table below which lists equivalent measurements.

To use the table to convert an amount in one system to the other system, you need to determine the conversion factor.

IMPERIAL TO METRIC EQUIVALENTS

IMPERIAL UNITS	METRIC UNITS
1 inch (in)	= 2.54 centimetres (cm)
1 foot (ft)	= .305 metres (m)
1 yard (yd)	= 0.914 m
1 mile (mi)	= 1.6 kilometre (km)
1 ounce (oz)	= 28.38 g
1 pound (lb)	= 454 grams (g) or 0.454 kg
1 quart (qt) Canadian	= 1.14 litres (L)



1 gallon (gal)	= 4.56 L
1 fluid ounce (fl oz)	= 28.4 ml
degrees Fahrenheit (F)	= $\frac{9}{5}$ degrees C + 32

To convert an amount in centimetres to an equivalent amount in inches, look at the Metric to Imperial side of the table. Find the centimetre to inch equivalent: 1 cm = 0.39 inches.

The number 0.39 in front of inches is the conversion factor. Multiply the original amount in centimetres by the conversion factor 0.39 and add the new unit, inches, to the answer. You have now converted an amount originally in centimetres to the equivalent amount in inches.

Suppose you are cutting a piece of block that is 4 feet long. When you are working from metric measurements on your reference table, you might want to convert this length to metres.

Refer to the Metric side of the table and find the conversion factor. The unit with 1 in front is the unit you are converting from. The unit with the conversion factor in front is the unit you are converting to.

Example Express 4mile in kilometres.

Solution

$$1\text{mile} = 1.6\text{km}$$

$$\begin{aligned}4\text{mi} &= 1.6\text{km}/\text{mi} \times 4\text{mi} \\ &= 6.4\text{km}\end{aligned}$$

So Boroko, a suburb in the National Capital District (NCD) is some 6.4km from a point of reference (Port Moresby), that is the reason why it is also known as 4mile. That is, Boroko is located at about 4mi from Port Moresby.

Likewise, for places like 3mile, 9mile and 17 mile in NCD their equivalent distances can be calculated using the examples above.

There other places in other towns in Papua New Guinea which are also known as 3mile and 4mile. The reference point can be the provincial administrative centre or the provincial shipping port.

**METRIC TO IMPERIAL EQUIVALENTS**

METRIC Units	IMPERIAL Units
1 cm	= 0.39 in
1 m	= 39.4 in = 3.3 ft = 1.09 yd
1 km	= 0.62 mi
1 g	= 0.035 oz = 0.0022 lb
1 kg	= 2.2 lb
1 L	= 0.88 qt
1 L	= 0.22 gal
Degree Celsius (C)	= $\frac{5}{9}(F - 32)^{\circ}$

Use the table as a conversion key.

Example Change 12 feet to metres.

Solution

Look at the Imperial to Metric chart to find the line that equates feet to metres. The conversion equivalent is 1 foot = 0.305 metres, so 0.305 is the conversion factor.

$12 \times 0.305 = 3.66$ multiply the original amount of 12 feet by the conversion factor 0.305
12 feet = 3.66 meters Place the metric unit, meters, after the answer.

To convert from an amount in an imperial unit to a metric unit, here are the steps:

1. Use the Metric to Imperial side of the table.
2. The conversion factor is in front of the metric unit.
3. Multiply the original imperial amount by the conversion factor.
4. Put the correct metric units after the multiplication answer.
5. The amount is now converted to metric.



Example Find the metric equivalent of 8 gallons.

Solution

Since you are converting an imperial amount, 8 gallons, to a metric unit, use the Imperial to Metric table. The table shows 1 gal = 4.56 L. The conversion factor is 4.56. Multiply 8 by 4.56 and then change the unit to litres.

$$\begin{aligned}1 \text{ gal} &= 4.56 \text{ L} \\8 \text{ gal} &= 4.56 \text{ L/gal} \times 8 \text{ gal} \\&= 36.48 \text{ L}\end{aligned}$$

If the table does not show a direct equivalent, you must convert within the original system until you have a unit with a metric conversion factor.

For example, if you want to convert feet to centimetres using the table on the previous page, you must first convert the feet to inches. Then you can use the conversion factor on the table for inches to centimetres. First convert the feet to inches then, convert inches to centimetres.

Example 1 Convert 5 feet to centimetres.

Solution

The tables do not show feet to centimetres, so first convert feet to inches. Use Table for Imperial Equivalents.

$$\begin{aligned}1 \text{ ft} &= 12 \text{ inches conversion factor is 12} \\5 \text{ ft} &= 5 \text{ ft} \times 12 \text{ in/ft} \\&= 60 \text{ in}\end{aligned}$$

Now convert 60 inches to centimetres. Use Table for Imperial to Metric Equivalents.

$$\begin{aligned}1 \text{ in} &= 2.54 \text{ cm conversion factor is 2.54} \\60 \text{ in} &= 2.54 \times 60 \\&= 152.4 \text{ cm}\end{aligned}$$

Example 2 Convert 5 feet into centimetres.

Solution

This time convert feet directly to meters and then convert the meters to centimetres.

$$\begin{aligned}1 \text{ ft} &= 0.305 \text{ m} \\5 \text{ ft} &= 5 \text{ ft} \times 0.305 \text{ m/ft} \\&= 1.525 \text{ m} \\&= 1.525 \text{ m} \times 100 \text{ cm/m} \\&= 152.5 \text{ cm}\end{aligned}$$



Notice the answers to the same question vary slightly in the examples. This is because the conversion factors have been rounded off. Do you still remember estimation error we discussed in the previous topics?

Example Suppose you drive 45 miles to work every day, what is the distance travelled in kilometres (km)?

Solution

$$\begin{aligned}\text{From the Imperial to Metric Table} \\ 1 \text{ mi} &= 1.6 \text{ km conversion factor is 1.6} \\ 45 \text{ mi} &= 1.6 \times 45 \\ &= 72 \text{ km}\end{aligned}$$

You might also encounter situations wherein you need to convert from metric to imperial units. Use the Metric to Imperial Equivalents.

The procedure for using the table is the same. The unit with the 1 in front is the metric unit. The number in front of the imperial unit is the conversion factor. Because there are fewer units in the metric system, there are sometimes several conversion factors after a metric unit.

For example you will see in the table that 1 meter can equal 39.4 inches or 3.3 feet or 1.09 yards.

To convert from metres to feet, choose the conversion factor that changes metres to feet, which is 3.3. To convert from metres to inches, choose the conversion factor 39.4.

To convert from a metric unit to an imperial unit, here are the steps.

1. The conversion factor is in front of the imperial unit in the Metric to Imperial Equivalent table.
2. Multiply the original metric amount by the conversion factor.
3. Put the correct imperial unit after the multiplication answer.
4. The amount is now converted to imperial units.

Example 1 You are travelling to a city 450 kilometres away. How many miles would you travel?

Solution

Since you are converting an amount, 450 km, in a metric unit to an imperial unit, use the Metric to Imperial chart. From the conversion table

$$\begin{aligned}1 \text{ km} &= 0.62 \text{ mi conversion factor is 0.62} \\ 450 \text{ km} &= 0.62 \times 450 \\ &= 279 \text{ mi}\end{aligned}$$



Example 2 Convert 4.5 litres to gallons.

Solution

From the Metric to Imperial table

1 L = .22 gal conversion factor is .22

$$\begin{aligned} 4.5 \text{ L} &= 4.5 \times .22 \\ &= .99 \text{ gal} \end{aligned}$$

Note:

1. When you convert, the first step is to check which system the amount is now in and which system you are converting to.
2. Next, choose the correct chart for that conversion.
3. Find the line with the required conversion factor on it.
4. Multiply the original amount by the conversion factor.
5. The multiplication answer is the amount in the new unit.
6. The last step is to write the new unit after the converted amount.

Example 3 A tree has a diameter of 5 inches. What is its diameter in millimetres?

Solution

The imperial to metric table has no inch to millimetre conversion factor, so we will convert in two steps.

1 in = 2.54 cm First convert from inches to centimetres (Use Table 3).

$$\begin{aligned} 5 \text{ in} &= 2.54 \times 5 \\ &= 12.7 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Since } 12.7 \text{ cm} &= 12.7 \times 10, \text{ then convert the answer to millimetres.} \\ &= 127 \text{ mm} \end{aligned}$$

Example 4 One litre of water weighs 1 kg. How many pounds does 4 L weigh?

Solution

There are two steps to this problem. First we need to find how much 4 L weighs in kilograms and then we need to convert the kilograms to pounds.

$$\begin{aligned} 1 \text{ L weighs } &1 \text{ kg} \\ 4 \text{ L weigh } &4 \text{ kg} \end{aligned}$$

$$\begin{aligned} 1 \text{ kg} &= 2.2 \text{ lb conversion factor is } 2.2 \\ 4 \text{ kg} &= 2.2 \times 4 \\ &= 8.8 \text{ lb} \end{aligned}$$



Example 5 A truck travels 30 mi/gal. How many km/L does it travel?

Solution

This conversion requires several steps.

1 mi = 1.6 km, first convert mi to km.

$$\begin{aligned} 30 \text{ mi} &= 1.6 \times 30 \\ &= 48 \text{ km} \end{aligned}$$

$$30 \text{ mi/gal} = 48 \text{ km/gal}$$

and

1 gal = 4.56 L Now convert gallons to litres.

$$48 \text{ km/gal} = 48 \text{ km}/4.56 \text{ L}$$

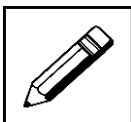
$48 \div 4.56 = 10.5 \text{ km/L}$ Divide 48 by 4.56 to find km/L.

$$30 \text{ mi/gal} = 10.5 \text{ km/L}$$

The truck travels 30 mi/gal or 10.5 km/L

Converting from one unit to another and from one system to another may sound difficult but it is not so. You just have to remember the steps we have discussed in this unit.

Revise this lesson and be ready to challenge yourself in the following learning activity.



LEARNING ACTIVITY 11.1.4.2



20 minutes

1) Convert the following to the indicated unit on the right after each blank.

- a) 15 m = _____ km
- b) 28.45 g = _____ kg
- c) 25, 000 m = _____ km
- d) 120 ft = _____ yd
- e) 38 L = _____ gal
- f) 190 mm = _____ in
- g) 280 km = _____ mi

2) Convert the following to the indicated unit on the right after each blank.

- a) 15 m = _____ km
- b) 28.45 g = _____ kg
- c) 25, 000 m = _____ km
- d) 120 ft = _____ yd
- e) 38 L = _____ gal
- f) 190 mm = _____ in
- g) 280 km = _____ mi



3) Convert the following to the indicated unit on the right after each blank.

- a) 15 m = _____ km
- b) 28.45 g = _____ kg
- c) 25, 000 m = _____ km
- d) 120 ft = _____ yd
- e) 38 L = _____ gal
- f) 190 mm = _____ in
- g) 280 km = _____ mi

4) Solve the following problems. Use the spaces for your working out.

- a) A 120 cm piece was cut from a rope measuring 30 m. What is the length in metres of the remaining piece?

Answer: _____

- b) Some selected tomatoes from the shop weigh an average of 50 g. approximately, how many pieces of tomatoes will there be in 2.5 kg?

Answer: _____

- c) John wants to transfer 5 L of milk in 1 gallon container. Is the container enough to hold the milk? Justify your answer.

Answer: _____



11.1.4.3 Solving, Plotting and Sketching Quadratic Equations

Given $y = 2x - 3$, is a linear equation. It is of the form $y = mx + c$, and the power of x is 1, thus may be referred to as an equation of the first degree. Quadratic Equation is a second degree equation such as $y = x^2$, and cubic is the third degree equation like $y = x^3$, and so forth.

Quadratic Equation is a second degree equation which is usually in the form $ax^2 + bx + c = 0$, where $a \neq 0$. In every quadratic equation, the numerical coefficient of the term with second degree (a) cannot be equal to zero. If and when $a = 0$ then the equation becomes linear.

The following equations are quadratic:

- | | | |
|------------------------|----------------------------------|---------------------------|
| 1. $y = ax^2$ | $a \neq 0$ and $b = 0, c = 0$ | e.g., $y = x^2$ |
| 2. $y = ax^2 + bx$ | $a \neq 0, b \neq 0$ and $c = 0$ | e.g., $y = -x^2 + 4x$ |
| 3. $y = ax^2 + c$ | $a \neq 0, c \neq 0$ and $b = 0$ | e.g., $y = 2x^2 - 18$ |
| 4. $y = ax^2 + bx + c$ | $a, b, c \neq 0$ | e.g., $y = x^2 - 3x + 10$ |

In type 3 ($y = ax^2 + c$) either the term a or c must have a minus sign (-) BUT not both, in order for the equation to have REAL roots. That is, the solution is a real number.

For example, $y = x^2 + 2$, $y = x^2 + 4$, $y = x^2 + 6$ are unsolvable. Also $y = -x^2 - 1$ and $y = -(x^2 + 1)$ and similar equations are unsolvable. But, $y = -x^2 + 4$ and $y = x^2 - 4$ are solvable, can be factorized and have real roots.

Solving Quadratic Equations can be dealt with using:

1. Table of values (when $y = 0$)
2. Factor Method
3. Completing the square Method
4. Quadratic Formula

For our purpose, we will use factor method and quadratic formula to solve and sketch. We may explore the completing – the – square method to show its relevance and significance, however, seldom use it because the process is quite long, but very convenient.

Factor Method

To be able to factorize quickly, you have to memorize and relate the quadratic equations to the form $x^2 + (a + b)x + ab = (x + a)(x + b)$ or the algebraic identities as given below. Where the middle term is the sum of the additive inverse of solutions, and the last term is the product of the additive inverse of the solution.



Given the binomial quadratic factors

$$\begin{array}{ll}
 (x + 3)(x + 2) \text{ their product will be } & x^2 + (3 + 2)x + 3 \times 2 & x^2 + 5x + 6 \\
 (x + 3)(x - 5) \text{ their product will be } & x^2 + (3 + -5)x + 3 \times -5 & x^2 - 2x - 15 \\
 (x - 3)(x + 1) \text{ their product will be } & x^2 + (-3 + 1)x + -3 \times 1 & x^2 - 2x - 3 \\
 (x - 3)(x - 2) \text{ their product will be } & x^2 + (-3 + -2)x + -3 \times -2 & x^2 - 5x + 6
 \end{array}$$

The simplified product is a trinomial expression. It is quadratic trinomial equation if the expression is equated to 0, such as $x^2 - 2x - 3 = 0$.

Identities

$$a^2 + 2ab + b^2 \equiv (a + b)^2$$

$$a^2 - 2ab + b^2 \equiv (a - b)^2$$

$$a^2 - b^2 \equiv (a + b)(a - b)$$

Examples

Product	Factors	Form
$x^2 + 10x + 21$ $x^2 - 4x - 21$ $x^2 + 4x - 21$ $x^2 - 10x + 21$	$(x + 3)(x + 7)$ $(x + 3)(x - 7)$ $(x - 3)(x + 7)$ $(x - 3)(x - 7)$	$x^2 + (a + b)x + ab = (x + a)(x + b)$
$x^2 + 6x + 9$ $x^2 - 8x + 16$ $3x^2 - 75$	$(x + 3)(x + 3)$ $(x - 4)(x - 4)$ $3(x + 5)(x - 5)$	$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$ $a^2 - b^2 = (a + b)(a - b)$

Factors Of Quadratics With Less Than Three Terms

Example 1 Factorize $y = 2x^2 - 2$

Solution

$$2x^2 - 2 = 0$$

$$2(x^2 - 1) = 0$$

$$2(x - 1)(x + 1) = 0$$

Factors are 2, $(x - 1)$ and $(x + 1)$

$$\text{or } 2x^2 - 2 = 0$$

$$2x^2 = 2$$

$$x^2 = 1$$

$$x = \sqrt{1} = \pm 1$$

Factors are $(x - 1)(x + 1)$, where 2 is lost.



Example 2 Factorize $y = 4x^2 + 2x$

Solution

$$4x^2 + 2x = 0$$
$$2x(2x + 1) = 0$$

Factors are $2x$ and $(2x + 1)$

Example 3 Factorize $y = 2x^2 - 8$

Solution

The equation is of the form $a^2 - b^2 \equiv (a + b)(a - b)$

$$2x^2 - 8 = 0$$
$$2(x^2 - 4) = 0$$
$$2(x + 2)(x - 2) = 0$$

Factors are 2 , $(x + 2)$ and $(x - 2)$

Alternatively we can equate as $2x^2 = 8$
 $x^2 = 4$
 $x = \pm 2$ [take square roots of both sides]

so the factors are $(x - 2)$ and $(x + 2)$, BUT we lost 2 the third factor. Use only when, finding roots(solutions) is important.

Discriminant

To test if it is possible to factorize and solve a quadratic equation $ax^2 + bx + c = 0$, we find the discriminant of the equation. It is denoted as $\Delta = b^2 - 4ac$. Now discriminant defines the following pattern,

$b^2 - 4ac > 0$	equation has two linear factors and two distinct real roots
$b^2 - 4ac = 0$	equation has two equal linear factors and two real roots
$b^2 - 4ac < 0$	equation has no linear factors and NO real roots

Having real roots means the solutions of x are real numbers or from real number field. If the roots are determined to be unreal, that is the discriminant is negative then they are **imaginary** roots.



Example 1 Determine if $2x^2 + 3x + 4 = 0$ has real roots.

Solution

$$\text{In } 2x^2 + 3x + 4 = 0, a = 2, b = 3, c = 4$$

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= 3^2 - 4 \times 2 \times 4 \\ &= 9 - 32 \\ &= -23\end{aligned}$$

$\Delta < 0$ therefore has no real roots.

Proof: $2x^2 + 3x + 4 = 0$ has no real roots

Solution by quadratic formula

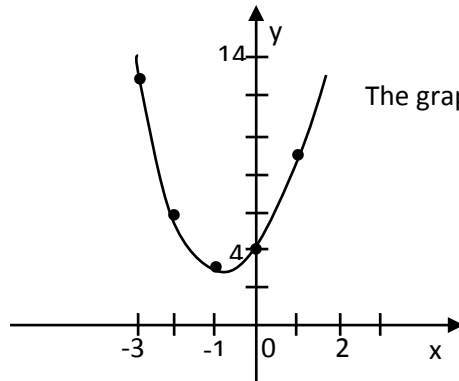
$$\begin{aligned}x &= \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times 4}}{2 \times 2} \\ &= \frac{-3 \pm \sqrt{9 - 32}}{4} \\ &= \frac{-3 \pm \sqrt{-23}}{4} \\ &= \frac{-3 - \sqrt{-23}}{4} \text{ or } \frac{-3 + \sqrt{-23}}{4} \\ &= \frac{-3 - \sqrt{23}i}{4} \text{ or } \frac{-3 + \sqrt{23}i}{4}\end{aligned}$$

Do not freak out seeing imaginary numbers; this is for the purpose of explanation of the importance of the discriminant.

The roots are **imaginary numbers** so the equation has **imaginary roots**. The radicand of negative 23 makes the number an imaginary number that is, we cannot plot on a **real** number line.



Such graphs (that have imaginary roots) **can be plotted** using table of values, but you will realize that the curve will **not intersect** x- axis. The curve will not intersect x- axis because the equation does not have real roots or real solutions.



The graph of $y = 2x^2 + 3x + 4$ does not intersect x-axis

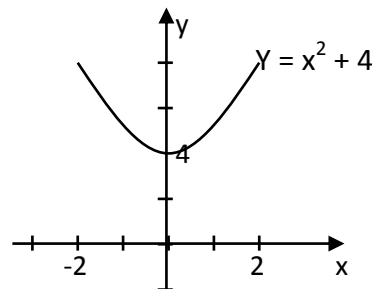
Example 2 Determine if $x^2 + 4 = 0$ has real roots.

Solution

$$\text{In } x^2 + 4 = 0, a = 1, b = 0, c = 4$$

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= 0^2 - 4 \times 1 \times 4 \\ &= 0 - 16 \\ &= -16\end{aligned}$$

$\Delta < 0$ therefore has no real roots.



Example 3 Determine if $6x^2 + x - 2 = 0$ has real roots.

Solution

$$\text{In } 6x^2 + x - 2 = 0, a = 6, b = 1, c = -2$$

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= 1^2 - 4 \times 6 \times -2 \\ &= 1 - -48 \\ &= 49\end{aligned}$$

$\Delta > 0$ therefore has two distinct real roots.

If you solve by any method, you will arrive at $x = \frac{1}{2}$ and $x = -\frac{2}{3}$

Example 4 Determine if $4 + 12x + 9x^2$ has real roots.

Solution

$$\text{In } 4 + 12x + 9x^2 = 9x^2 + 12x + 4,$$

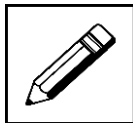
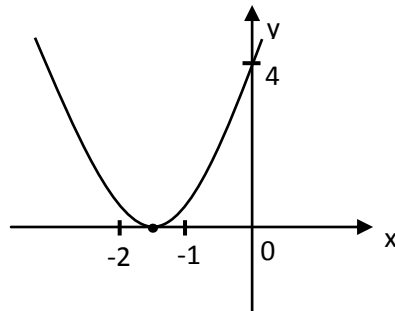
$$9x^2 + 12x + 4 = 0, \text{ equate to zero in-order to solve } a = 9, b = 12, c = 4$$



$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= 12^2 - 4 \times 9 \times 4 \\ &= 144 - 144 \\ &= 0\end{aligned}$$

$\Delta = 0$ therefore has two equal real roots.

If you solve by any method, you will arrive at $x = \frac{-3}{2}$ and $x = \frac{-3}{2}$. For such graphs, the x-axis is touched at that point, that is the solution is the point of tangency of the curve at a-axis.



STUDENT LEARNING ACTIVITY 11.1.4.3.1



20 minutes

1. Find the products of the following

- (a) $(x + 3)(x + 9) =$
- (b) $(x + 3)(x - 6) =$
- (c) $(x + 6)(x - 4) =$
- (d) $(x - 2)(x - 11) =$
- (e) $(2x - 3)(x + 6) =$
- (f) $(3x + 4)(x - 3) =$
- (g) $(1 - 4x)(2 + 5x) =$
- (h) $(6 - x)(2x + 5) =$

2. Factorize the following quadratic equations.

a) $3x^2 - 12 = 0$



b) $y = x^2 + 3x$

c) $x^2 - 8x = 0$

d) $y = x^2 - 100$

3. Use the identity to find factors of the following quadratic expressions.

a) $3x^2 - 12x + 12$

b) $x^2 + 14x + 49$

c) $x^2 - 144$

d) $x^2 - 2x - 80$

4. Factorize and solve

a) $6x^2 - 12x + 6 = 0$

b) $x^2 + 3x - 88 = 0$

c) $4x^2 - x = 0$

d) $x^2 - 20x + 100 = 0$



e) $x^2 - 225 = 0$

f) $6x^2 - 3x = 0$

g) $x^2 - 6x + 9 = 0$

h) $x^2 + 18x + 81 = 0$

i) $2x^2 + 12x + 18 = 0$

5. Use discriminant to determine if the equation has two distinct real roots or no real roots.

a) $2x^2 - 7x - 4 = 0$

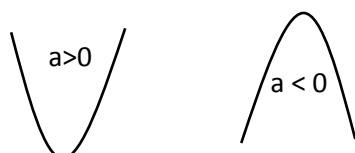
b) $16x^2 + 4x - 2 = 0$

c) $2x^2 - 3x + 4 = 0$

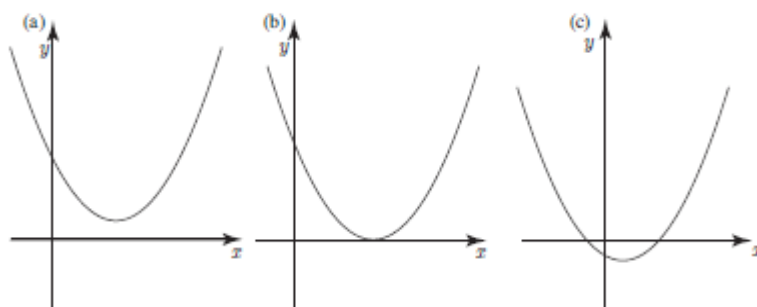
d) $-9x^2 + 6x - 1 = 0$

**Graphing the function $y = ax^2 + bx + c$, $y = f(x)$**

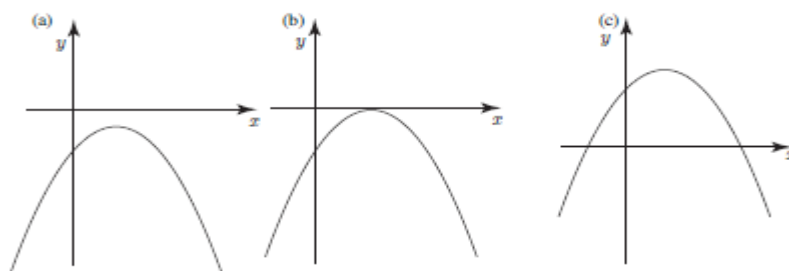
If the numerical coefficient of x^2 is positive ($a > 0$), the parabola opens upward while if it is negative ($a < 0$), the parabola opens downwards.



As the positive value of a increases, the graph moves as in the figure below



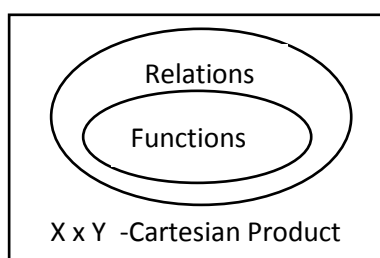
While when the negative value of a changes, this is how the graph moves



The given quadratic equation in the form $ax^2 + bx + c = 0$, can be transformed into a quadratic function where $y = f(x)$. That is, the value of y depends on the variability of x . As the x value changes, y value also changes. Every value of x is mapped to only one value of y .

A function is a relation in which each first element of an ordered pair has only one second element.

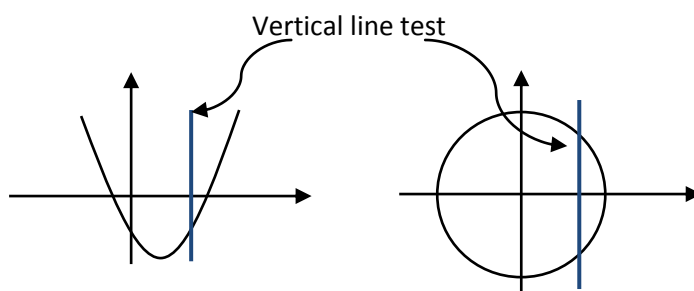
The relation $x \rightarrow x^2$ is a function and we therefore write $f: x \rightarrow x^2$, which is read as the function of x is x^2 . If we write $y = f(x)$ we mean that y is a function of x .





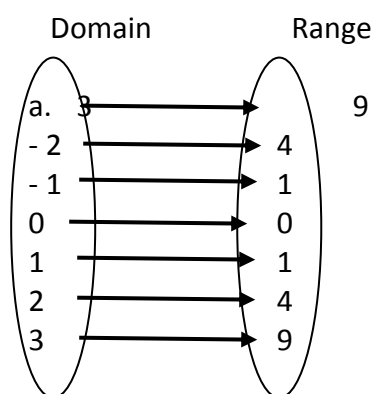
A vertical line test can be used on a graph to test if the function is a function. Say, for $\{(x, y) : y = x^2\}$ and $\{(x, y) : x^2 + y^2 = 4\}$ are a quadratic and a circle. A quadratic is a function and the circle is NOT a function.

Graphs of $y = x^2 + 2x - 3$ and $x^2 + y^2 = 9$



The domain is the set of x values, and the range is the set of y values. Domains are independent variables. Ranges are dependent variables, their values depend on the domain or x values.

$$f(x) = x^2$$



The **Quadratic Function** is written in the form

$$y = ax^2 + bx + c$$

where: a, b and c are real numbers, $a \neq 0$

The graph of a quadratic function is a **parabola**. It is a smooth curve with an axis of symmetry with its vertex (turning point).

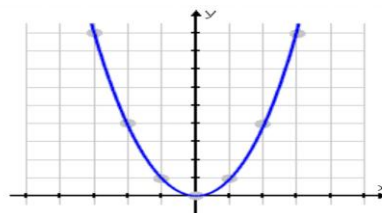
Graph of $y = x^2$

Consider the relation $x \rightarrow x^2$ with the domain $\{-3, -2, -1, 0, 1, 2, 3\}$. The range is found by substituting each element of the domain into x^2 . Thus the table of value will be as:

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9



Hence the range is the set $\{0, 1, 4, 9\}$.



In this graph, the axis of symmetry is the y-axis. The vertex of the parabola is at $(0, 0)$ or the origin.

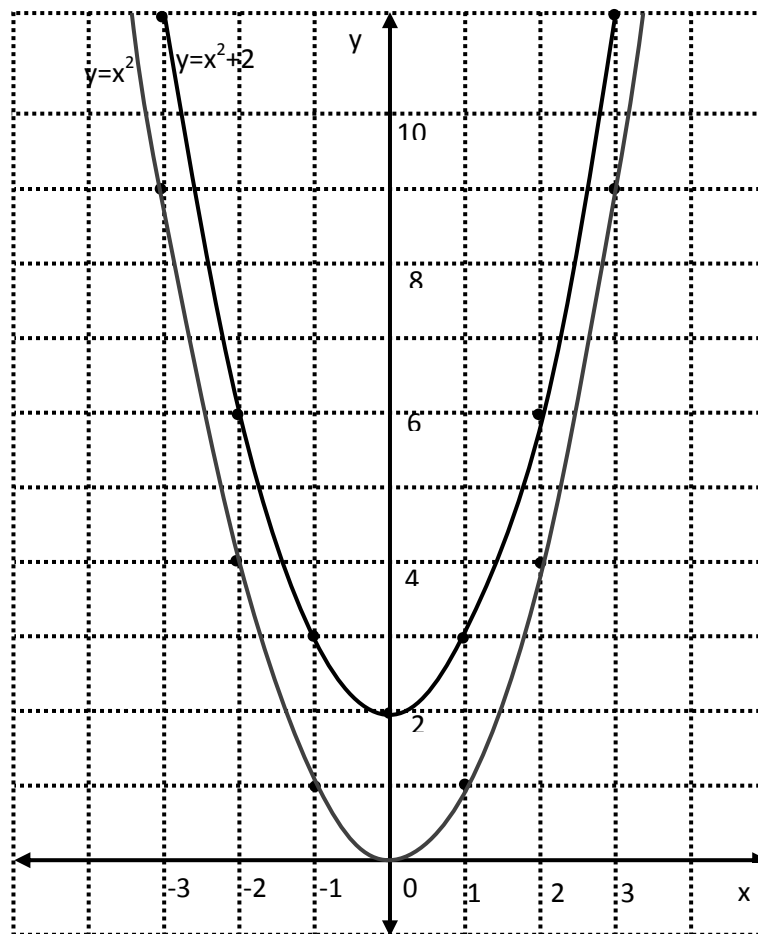
Graph of $y = x^2 + 2$

x	-3	-2	-1	0	1	2	3
y	11	6	3	2	3	6	11

This graph has the domain $x: -3 \leq x \leq 3$ and the range $y: 2 \leq y \leq 11$. The domain is read as 'x is such that -3 is less than or equal to x which is less than or equal to 3. That means the domain consists of the numbers from -3 to 3 as depicted in the table of values above.

Observe that when a constant 2 is added to the function $y = x^2$, the graph shifts 2 units upward. Its vertex is now at $(0, 2)$. The axis of symmetry remains the same however, it can be noticed that the graph is getting narrower (dilation).

Similarly, if a negative constant (c) is added, the graph will shift downward and it will become wider.



Plotting Quadratic Graphs

Steps

1. Solve the equation
2. Set the domain to at least 2 units above x_1 , and at least 2 units below x_2
3. Complete the table of values
4. Rule axes
5. Set the scale
6. Plot the points obtained in the table of values
7. Join the points with a curve



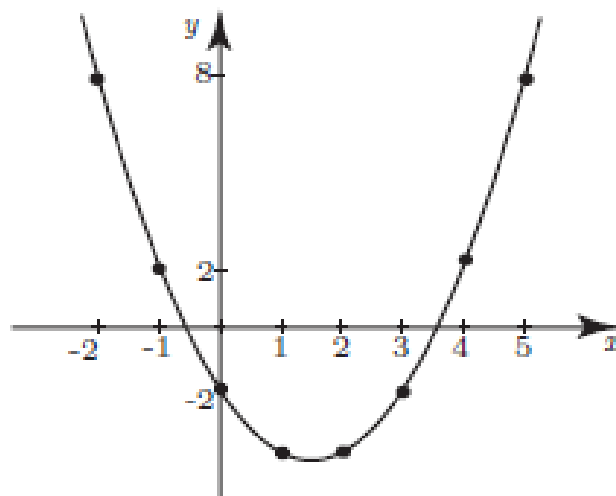
Example 1 Graph the function $y = x^2 - 3x - 2$

Solution

1. Roots $x^2 - 3x - 2 = 0$
 $(x - 1)(x - 2) = 0$ when $x - 1 = 0$, $x = 1$
2. Domain $\{-2, -3, \dots, 5\}$
3. Table of values

x	-2	-1	0	1	2	3	4	5
y	8	2	-2	-4	-4	-2	2	8

4. Rule and label axes on your graph paper.
5. Lay out the scale: x-axis 1:1 and y-axis 1:2
6. From the table of values, the graph is plotted as follows



7. Join the points with a curve.



Sketching a Quadratic Graph

In sketching we illustrate key features of the graph. In plotting, we find and plot all the points within the given domain and range.

To sketch a quadratic curve, determine the following:

- | | |
|---|----------------|
| 1. The two roots (solutions of x) of the curve | $x = a, x = b$ |
| 2. The y -intercept (at $x = 0$) | $y = c$ |
| 3. The axis of symmetry | $x = -b/2a$ |
| 4. The Vertex or Turning Point (TP) | (x, y) |

Then rule the axes and illustrate all four properties above to give the main impression of the quadratic graph.

Example 1 Sketch the graph of $y = x^2 - 2x - 8$

Solution

Roots $x^2 - 2x - 8 = 0$
 $(x - 4)(x + 2) = 0$

when $x - 4 = 0$, $x = 4$
when $x + 2 = 0$, $x = -2$

y – intercept at $x = 0$ is $(0, -8)$ Can be read from equation, $a = 1$, $b = -2$ and $c = -8$

Proof: $y = (0)^2 - 2(0) - 8 = 0 - 0 - 8 = -8$

Axis of symmetry $x = -b/2a$ where $a = 1$, $b = -2$

$$\begin{aligned}x &= \frac{-(-2)}{2 \times 1} \\&= \frac{2}{2} \\&= 1\end{aligned}$$

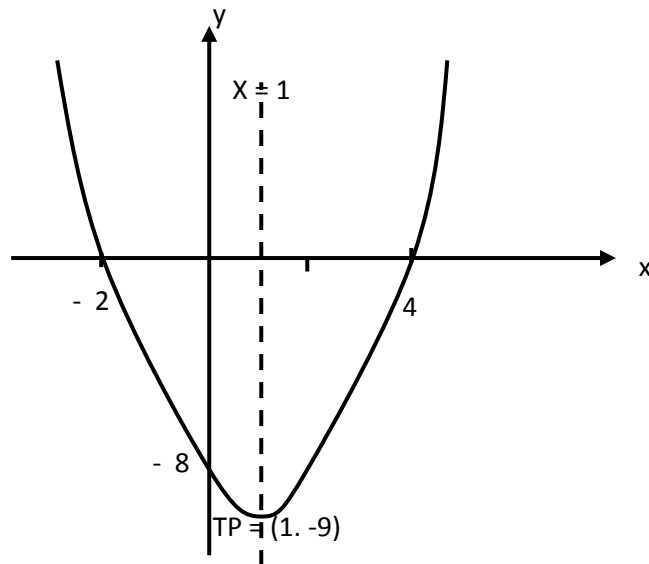
Turning Point of $y = x^2 - 2x - 8$ when $x = 1$

$$\begin{aligned}Y &= (1)^2 - 2(1) - 8 \\&= 1 - 2 - 8 \\&= -9\end{aligned}$$

TP = $(1, -9)$ Minimum



Sketch of the graph of $y = x^2 - 2x - 8$



Example 2 Sketch the graph of $y = -x^2 + 10x - 21$ showing all the necessary features.

Solution

Roots

$$\begin{aligned} -x^2 + 10x - 21 &= 0 \\ -(x^2 - 10x + 21) &= 0 \\ -1(x - 3)(x - 7) &= 0 \end{aligned}$$

$$\begin{aligned} \text{when } x - 3 &= 0, & x &= 3 \\ \text{when } x - 7 &= 0, & x &= 7 \end{aligned}$$

y – intercept at $x = 0$ is $(0, -21)$ Can be read from equation, $a = -1$, $b = 10$ and $c = -21$

Axis of symmetry

$$\begin{aligned} x &= -b/2a \\ x &= \frac{-10}{2(-1)} \\ &= \frac{-10}{-2} \\ &= 5 \end{aligned}$$

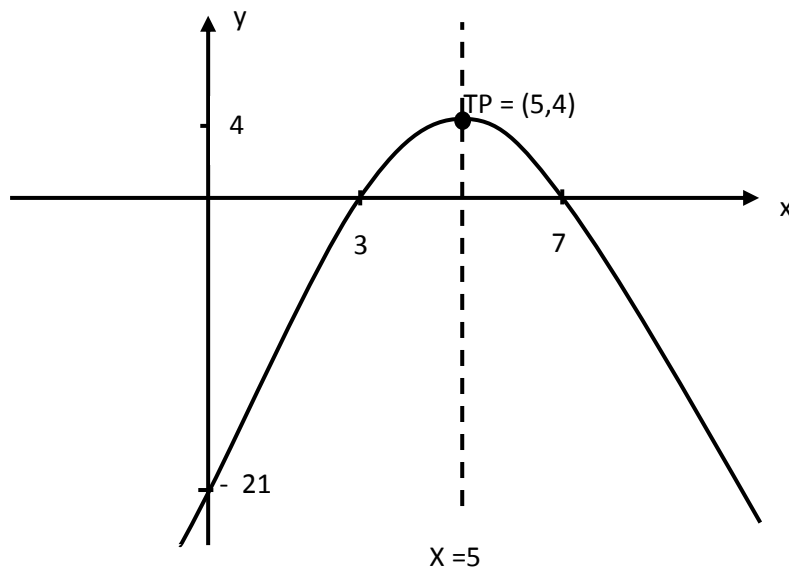
Turning Point of $y = -x^2 + 10x - 21$ when $x = 5$

$$\begin{aligned} Y &= -(5)^2 + 10(5) - 21 \\ &= -25 + 50 - 21 \\ &= 4 \end{aligned}$$

TP = $(5, 4)$ Maximum



Sketch of the graph of $y = -x^2 + 10x - 21$



When $a < 0$ or negative, the quadratic curve has a maximum point (TP); when $a > 0$ or positive the curve has a minimum point (TP).

Form $y = a(x - h)^2 + k$

When a quadratic equation is expressed in the form $y = a(x - h)^2 + k$, by application of squaring in order to factorize, the axis of symmetry ($x = h$) and the vertex or TP can easily be read out. The value of (h, k) is the vertex.

The transformation of the equation does not change the value of the original equation. If we reverse the operation, we will arrive at the original equation of the form $ax^2 + bx + c$.

Example 1 What is the axis of symmetry and the vertex of the curve $y = x^2 + 4x - 12$?

Solution

Graph of $y = x^2 + 4x - 12$ is of the form $ax^2 + bx + c$

$$x^2 + 4x - 12$$

$$(x^2 + 4x) - 12$$

$$(x^2 + 4x + 4) - 12 - 4$$

$$(x + 2)(x + 2) - 16$$

$$(x + 2)^2 - 16$$

$$h = -2, k = -16$$

Group first two terms

Mult. 4 by $\frac{1}{2}$, add its square

Factorize group, simplify terms

Note: $-h = 2$

Axis of symmetry at $x = -2$, vertex $(-2, -16)$



Example 2 Find the vertex of the curve $y = 2x^2 - 3x - 2$

Solution

Graph of $y = 2x^2 - 3x - 2$ is of the form $ax^2 + bx + c$

$$2x^2 - 3x - 2$$

$$(2x^2 - 3x) - 2$$

$$2(x^2 - \frac{3}{2}x) - 2$$

$$2(x^2 - \frac{3}{2}x + (\frac{3}{4})^2) - 2 - (\frac{3}{4})^2$$

$$2(x - \frac{3}{4})^2 - 2 - \frac{9}{16}$$

$$2(x - \frac{3}{4})^2 - 2\frac{9}{16}$$

$$h = \frac{3}{4}, k = -2\frac{9}{16}$$

Group first two terms

Factorize the group

Mult. $\frac{3}{2}$ by $\frac{1}{2}$ and add its square

Factorize group, simplify

Read (h,k)

Axis of symmetry at $x = \frac{3}{4}$, vertex $(\frac{3}{4}, -2\frac{9}{16})$

However, in some cases, the use of finding the roots by using the quadratic formula is found to be more convenient, especially when $a \neq 1$ in $ax^2 + bx + c = 0$.

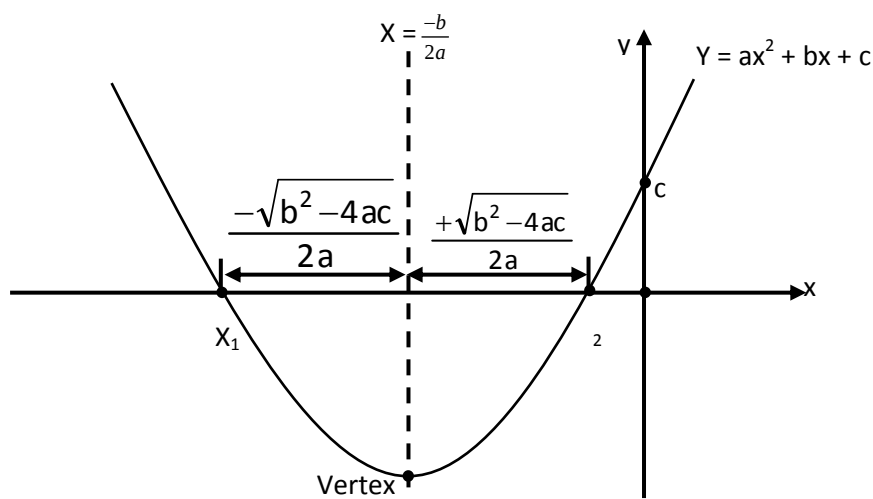
Quadratic formula

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \text{ or } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula was derived by completing the square on $ax^2 + bx + c = 0$. This is shown here after, in completing the square.

In the formula, $x = \frac{-b}{2a}$ gives the axis of symmetry. And adding $\frac{-\sqrt{b^2 - 4ac}}{2a}$ yields the root, which is the distance to the left of axis of symmetry along the x-axis, whilst adding $\frac{+\sqrt{b^2 - 4ac}}{2a}$ yields the root, which is the distance to the right of the axis of symmetry along the x-axis.

The roots are points where the curve intersects x – axis (solutions).





Example 1 Solve for x in $y = 3x^2 - 10x + 3$

Solution

First, equate to zero. That is when $y = 0$, $3x^2 - 10x + 3 = 0$.

Now we can observe that $a = 3$, $b = -10$, $c = 3$

Substitute into the formula

$$\begin{aligned}
 x &= \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} & \text{or} & & x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 x &= \frac{-(-10)}{2 \times 3} \pm \frac{\sqrt{(-10)^2 - 4 \times 3 \times 3}}{2 \times 3} & & & x &= \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \times 3 \times 3}}{2 \times 3} \\
 &= \frac{10}{6} \pm \frac{\sqrt{100 - 36}}{6} & & & &= \frac{10 \pm \sqrt{100 - 36}}{6} \\
 &= \frac{10}{6} \pm \frac{\sqrt{64}}{6} & & & &= \frac{10 \pm \sqrt{64}}{6} \\
 &= \frac{10}{6} \pm \frac{8}{6} & & & &= \frac{10 \pm 8}{6} \\
 &= \frac{10}{6} + \frac{8}{6} \text{ or } \frac{10}{6} - \frac{8}{6} & & & &= \frac{18}{6} \text{ or } \frac{2}{6} \\
 &= \frac{18}{6} \text{ or } \frac{2}{6} & & & &= 3 \text{ or } \frac{1}{3} \\
 &= 3 \text{ or } \frac{1}{3} & & & & x = 3, x = \frac{1}{3}
 \end{aligned}$$

Therefore the roots of $3x^2 - 10x + 3 = 0$ are $x = 3$ and $x = 1/3$.

Example of the application of either form of formula is provided for you above. Use the quadratic formula whenever you cannot easily factorize and solve.

Then perform steps 2, 3 and 4 before you draw a sketch of the quadratic graph.

y – intercept (0, 3) Read from equation, $a = 3$, $b = -10$ and $c = 3$

Axis of symmetry $x = -b/2a$

$$x = \frac{-(-10)}{2 \times 3}$$

$$= \frac{10}{6}$$

$$= \frac{5}{3}$$

$$= 1\frac{2}{3}$$

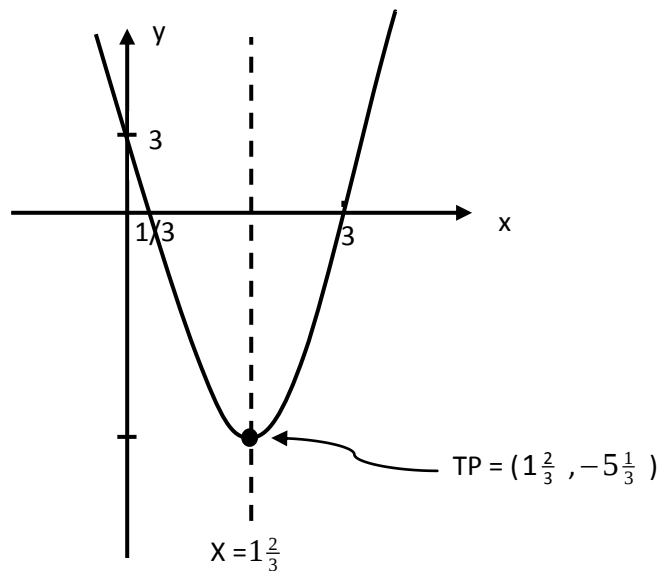


Turning Point of $y = 3x^2 - 10x + 3$ when $x = 1\frac{2}{3}$ or $5/3$

$$\begin{aligned} Y &= 3(5/3)^2 - 10(5/3) + 3 \\ &= 3(25/9) - 50/3 + 3 \\ &= 25/3 - 50/3 + 3 \\ &= -25/3 + 9/3 \\ &= -16/3 \\ &= -5\frac{1}{3} \end{aligned}$$

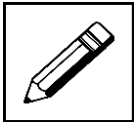
$$TP = (1\frac{2}{3}, -5\frac{1}{3})$$

It is now possible to sketch graph of $y = 3x^2 - 10x + 3$ as given below.



Ensure that whenever you sketch, the curve should be made to seem symmetrical about the axis of symmetry.

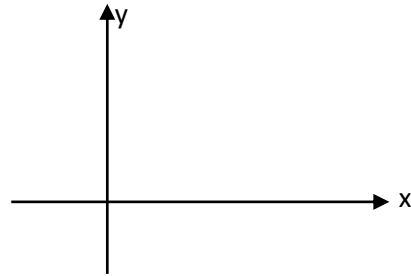
Always use free hand to draw the curve. NEVER use a ruler.

**STUDENT LEARNING ACTIVITY 11.1.4.3.2**

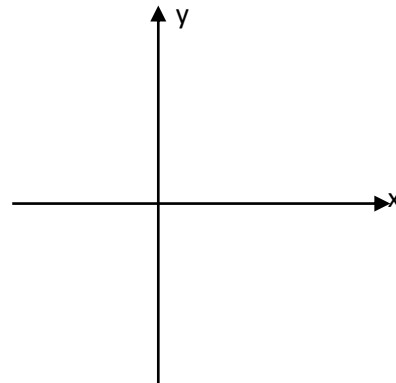
20 minutes

1. Sketch the graphs of the following on the axes provided.

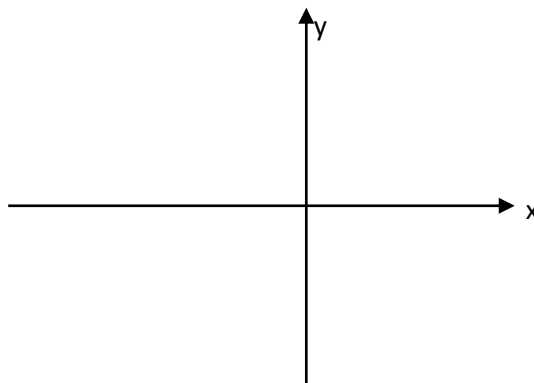
a) $y = x^2 - 8x + 15$



b) $y = x^2 - 2x - 15$



c) $y = x^2 + 2x - 15$



2. Express the following equation in the form $y = a(x - h)^2 + k$

a) $y = 2x^2 - 10x + 30$



b) $y = 4x^2 + 5x - 6$

c) $y = x^2 + 10x - 11$

3. Use the quadratic formula to solve for x.

a) $Y = 2x^2 + 3x - 5$

b) $Y = 3x^2 + 4x - 4$

c) $2 - 3x - 2x^2 = 0$

d) $-6x^2 + x + 2 = 0$



Completing the Square Method

The quadratic formula is derived by completing the square on $ax^2 + bx + c = 0$

Study the steps laid out on $ax^2 + bx + c = 0$ to derive the formula.

Steps

1. Divide each of the three terms on the LHS by coefficient a
2. Eliminate c/a from the LHS (appears on the RHS)
3. To be able to factorize LHS, multiply b/a by $\frac{1}{2}$, and add its square
4. Factorize LHS, simplify the RHS
5. Solve for x

Let us complete the square on $ax^2 + bx + c = 0$.

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad \text{divide each term by } a$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \quad \text{remove } \frac{c}{a} \text{ from LHS}$$

$$x^2 + \frac{b}{2a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 \quad \text{we multiply } \frac{b}{a} \text{ by a half and add its square}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} \quad \text{factorize LHS, expand and simplify the RHS}$$

$$= \frac{b^2 - 4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right) = \pm \frac{\sqrt{b^2 - 4ac}}{2a} \quad \text{take the root of both sides}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \quad \text{solve for } x$$



Example Complete the square to solve for x in $8x^2 - 2x - 1 = 0$

Solution

$$8x^2 - 2x - 1 = 0$$

$$x^2 - \frac{2}{8}x - \frac{1}{8} = 0$$

$$x^2 - \frac{2}{8}x = \frac{1}{8}$$

$$x^2 - \frac{1}{8}x + \left(\frac{1}{8}\right)^2 = \frac{1}{8} + \frac{1}{64}$$

$$\left(x - \frac{1}{8}\right)^2 = \frac{8+1}{64}$$

$$= \frac{9}{64}$$

$$x - \frac{1}{8} = \pm \frac{3}{8}$$

$$x = \frac{1}{8} + \frac{3}{8} \text{ or } \frac{1}{8} - \frac{3}{8}$$

$$x = \frac{1}{2} \text{ or } -\frac{1}{4}$$

• Divide by a

• Eliminate c/a from LHS

• $b/a \times \frac{1}{2}$, add its square to both sides

• simplify RHS

• Take the square root

• Solve for x

Using Square Root Method to Solve

Apart from the quadratic formula, the **square root method** can also be used only when the equation is as given in Example 1 below, or if you have the skill to express as below.

Example 1 Solve for x in $(x+3)^2 = 49$

Solution

Get the square root of both sides of the equation : $\sqrt{(x+3)^2} = \sqrt{49}$
 $(x+3) = \pm 7$

Then:

$$\begin{array}{ll} (x+3) = 7 & ; \quad (x+3) = -7 \\ x = 7 - 3 & ; \quad x = -7 - 3 \\ x = 4 & ; \quad x = -10 \end{array}$$

Therefore the roots of $(x+3)^2 = 49$ are 4 and -10.

Proof of the solution by factor method

$(x+3)^2 = 49$	Express in the form $ax^2 + bx + c = 0$
$x^2 + 3x + 3x + 9 = 49$	Squaring LHS
$x^2 + 6x + 9 - 49 = 0$	Simplify LHS, eliminate 49 from RHS
$x^2 + 6x - 40 = 0$	Simplify LHS
$(x + 10)(x - 4) = 0$	Factorize
$x = -10, x = 4$	Solve

The solutions are the same.



Example 2 Find the roots in $x^2 + 8x + 16 = 36$

In this example, the left side of the equation can be factored:

$$x^2 + 8x + 16 = 36$$

$$(x+4)(x+4) = 36$$

$$(x+4)^2 = 36$$

Since both sides of the equation are perfect square:

$$\sqrt{(x+4)^2} = \sqrt{36}$$

Then:

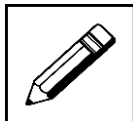
$$(x+4) = \pm 6$$

To get the roots:

$$\begin{array}{ll} x+4 = 6 & ; \quad x+4 = -6 \\ x = 6 - 4 & ; \quad x = -6 - 4 \\ x = 2 & ; \quad x = -10 \end{array}$$

Therefore the roots of $x^2 + 8x + 16 = 36$ are 2 and -10.

For your practice, you may use the quadratic formula or factor method to prove that the solutions are correct.



LEARNING ACTIVITY 11.1.4.3.3



20 minutes

1. Factorize and solve.

(a) $x^2 - 6x + 8 = 0$

(b) $x^2 + 9x + 8 = 0$

(c) $x^2 - 9 = 0$

(d) $x^2 + 3x - 10 = 0$

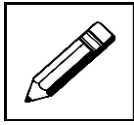
2. Find the roots of the following by completing –the-square method.

a. $x^2 - 7x + 12 = 0$

b. $x^2 + 2x - 8 = 0$

c. $2x^2 + 5x - 3 = 0$

d. $3x^2 + 10x + 8 = 0$

**STUDENT LEARNING ACTIVITY 11.1.4.3**

20 minutes

1. Use the discriminant to check if the following have real roots.

(a) $x^2 - 8x + 16 = 0$

(b) $x^2 - 13x + 30 = 0$

(c) $2x^2 - 5x - 3 = 0$

(d) $x^2 + 2x + 5 = 0$

(e) $2x^2 - 12x + 18$

2. Find the roots of the following using any method.

a) $x^2 = 64$

b) $3x^2 + 75 = 0$

c) $(x+3)^2 = 20$

d) $x^2 + 12x + 36 = 0$



3. Sketch the following quadratic functions.

a) $y = x^2 - x - 6$

b) $y = -x^2 + 4x - 3$

4. Complete the square to solve for x in

a) $2x^2 + 11x + 12 = 0$

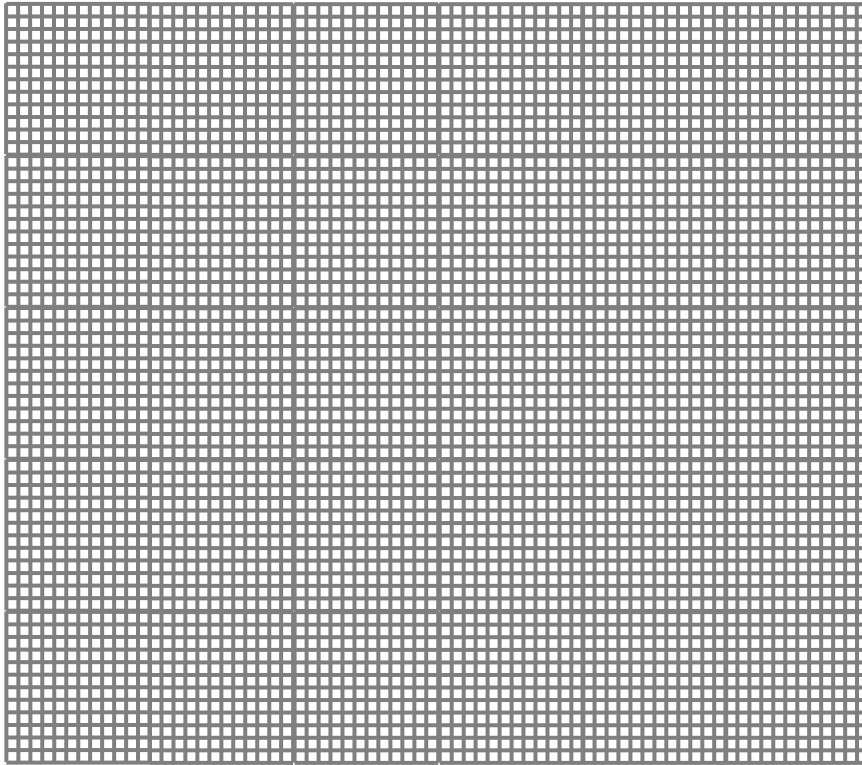
b) $y = x^2 - 4x - 12$

c) $2x^2 + 5x - 12 = 0$

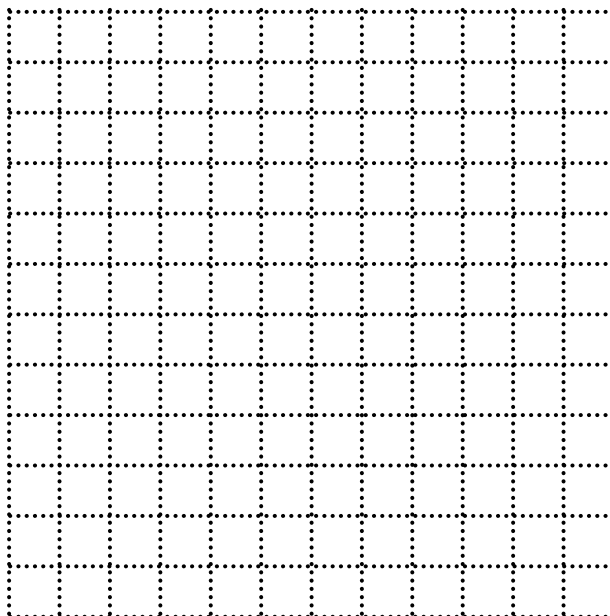


5. Graph the following quadratic functions.

a) $y = x^2 - x - 12$



b) $y = -x^2 - 4x$





6. What are axis of symmetry and vertex of the curve $y = x^2 - 8x + 15$

7. Calculate the vertex of the curve $y = 2x^2 + 5x + 2$



11.1.4.4 Inequalities

An inequality involves one of the four symbols $>$, $<$, $\geq$, $\leq$.

The following statements illustrate the meaning of each symbol.

$x > 3$	x is greater than 3	4,5,6...
$x < 3$	x is less than 3	2,1,0...
$x \geq 3$	x is greater than or equal to 3	3,4,5...
$x \leq 3$	x is less than or equal to 3	3,2,1...

Inequalities can be represented on a number line.

Example 1 Represent the following inequalities on the number line.

a) $x \geq 2$

b) $x < -1$

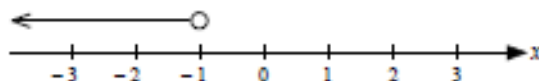
c) $-2 < x \leq 4$

Solution

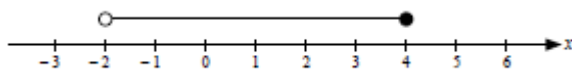
a) In the inequality $x \geq 2$, it means that the values of x includes 2,3 4 up to positive infinity. A dot is used to represent the start of this inequality.



b) The inequality $x < -1$ means that the values of x does not include -1, therefore a hollow dot is used.



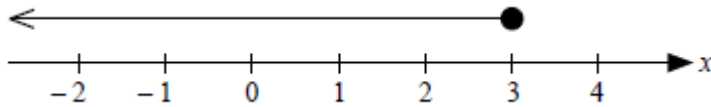
d) In the inequality $-2 < x \leq 4$, the values of x ranges from numbers more than -2 to 4. Therefore dots on both ends of the line were used.



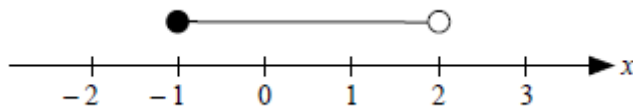


Example 2 Write the inequality to describe the region represented by the following lines.

a)



b)



Solution

a) The diagram indicates that the values of x must be less than or equal to 3 which can be written as $x \leq 3$.

b) The diagram indicates that x must be greater than or equal to -1 and less than 2. This is written as $-1 \leq x < 2$.

Solutions of Linear Inequalities

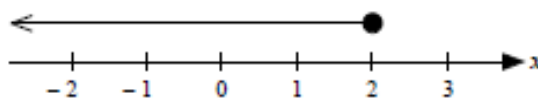
Solving for the solutions of Linear inequalities maybe compared to the way we deal with linear equations. But in this case aside from the changes in the sign, the direction of the symbols must be noted and considered.

Example 3 Solve the inequality $6x - 7 \leq 5$

Solution

$6x - 7 \leq 5$	Begin with the given
$6x \leq 12$	Add 7 on both sides (or transpose 7)
$x \leq 2$	Divide both sides by 6

This inequality can be represented by the number line below





Example 4 Solve the inequality $4(x - 2) > 20$

Solution

$4(x - 2) > 20$	Begin with the given
$x - 2 > 5$	Divide both sides by 4
$x > 7$	Add 2 on both sides (transpose 2)

Example 5 Solve the inequality $5 - 6x \geq -19$

Solution

$5 - 6x \geq -19$	Begin with the given
$5 \geq -19 + 6x$	Note that the inequality contains $-6x$, therefore transpose the term $6x$ on the other side by adding $6x$ on both sides.
$24 \geq 6x$	Add 19 on both sides
$4 \geq x$	Divide both sides by 6

The answer can also be written as $x \leq 4$.

$5 - 6x \geq -19$	Begin with the given
$-6x \geq -19 - 5$	
$-6x \geq -24$	
$\frac{-6x}{-6} \geq \frac{-24}{-6}$	
$x \leq 4$	Change the direction of the sign when multiplying or dividing by a negative number.

Unlike equalities, you can just interchange the values on both sides, in inequalities as you interchange you have to use the opposite symbol.

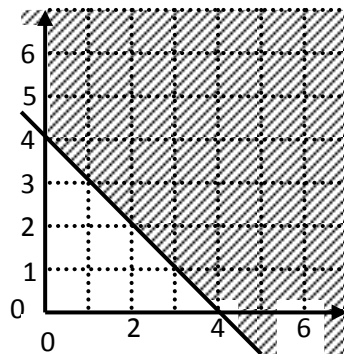
The inequality sign changes when quantities are multiplied or divided by a negative number.

Systems of Inequality and Regions

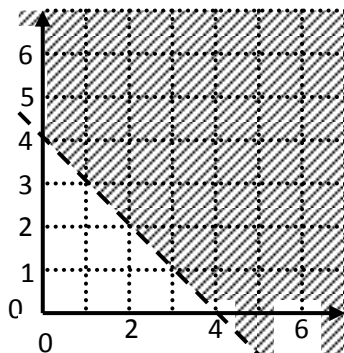
When an inequality involves two variables, it can be represented by a region (shaded part) on a graph.

In Analytic Geometry, when a straight line is ruled in any direction, the straight line divides the plane in two equal parts.

The inequality $x + y \geq 4$ can be illustrated in the graph as



Notice that the line is like the graph of the equation $x + y = 4$ as it uses the coordinates of the equation. The region above the graph or the shaded area represents the inequality $x + y \geq 4$. The inequality $x + y > 4$ can be illustrated in the graph as



What is the difference between this graph and the one on previous page? The straight line.

When the symbols $\leq$ and $\geq$ are used, the line is a solid line while when the symbols are $>$ and $<$ we use a broken or cut line.

If the inequality has $\geq$ or $\leq$ use a straight line. If the inequality has $>$ or $<$ use a cut-line or a dash.

Example 5 Show the region (by shading) that satisfies that inequality $y \geq 4x - 7$

Solution

The region has a boundary on the line $y = 4x - 7$. Remember, as you graph the line, change the inequality symbol to $=$ sign to determine the boundary or the line.

Use the table below to graph the line $y = 4x - 7$

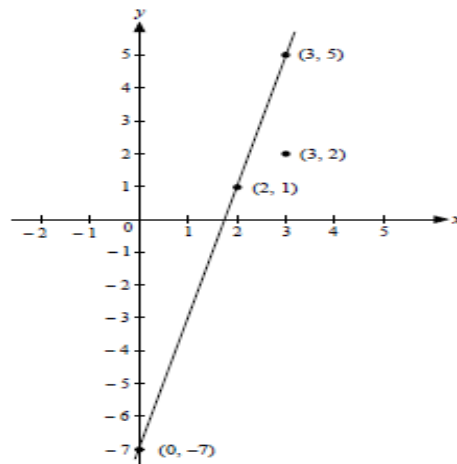
x	3	2	1	0
y	5	1	-3	-7



Plot the points and draw the line.

Note that a solid line is used because the inequality symbol in this example is $\geq$.

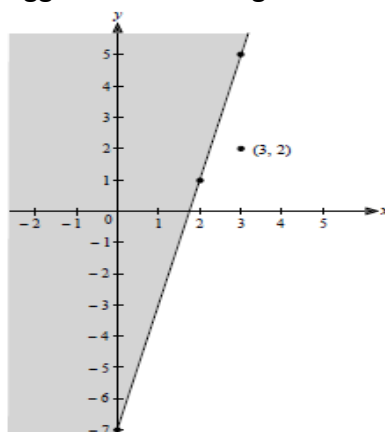
To determine to which region is shaded, name a point on the graph which does not belong to the line (non- collinear point). In this example we use the point (3, 2) as shown on the graph.



If the values of $x = 3$ and $y = 2$ are substituted into the inequality

$$\begin{aligned}y &\geq 4x - 7 \\2 &\geq 4(3) - 7 \\2 &\geq 5 \text{ FALSE}\end{aligned}$$

This is FALSE statement. Therefore, it will also be false (or not true) to any other point on the right side of the line. Thus, it suggests that the region to be shaded is the left side of the line.



When the statement holds TRUE, shade the region containing the point. If FALSE shade the region on the other side of the straight line.

**Solving Inequalities and Sketching Regions**

A straight line divides the plane into two equal regions.

Example 6 Show the region (by shading) that satisfies that inequalities.

$$y \geq 2x - 1$$

$$y < -2x + 3$$

Solution

Intersection points $y \geq 4x - 1$

When $y = 0$, $0 = 4x - 1$, therefore $x = 1/4$ Point $(1/4, 0)$

When $x = 0$, $y = 4 \cdot 0 - 1$, therefore $y = -1$ Point $(0, -1)$

Region to shade

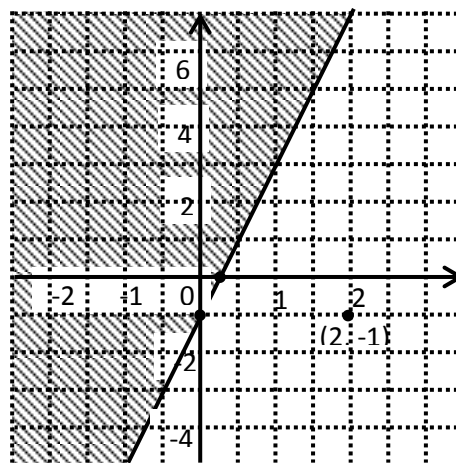
$y \geq 4x - 1$ Test Point $(2, -1)$

$$-1 \geq 4 \cdot 2 - 1$$

$$-1 \geq 8 - 1$$

$$-1 \geq 7 \text{ is False}$$

Shade region that does not contain the test point $(2, -1)$



Intersection points $y < -2x + 3$

When $y = 0$, $0 = -2x + 3$, therefore $x = 3/2$ Point $(1\frac{1}{2}, 0)$

When $x = 0$, $y = -2 \cdot 0 + 3$, therefore $y = 3$ Point $(0, 3)$

Region to shade

$y < -2x + 3$ Test Point $(1, 2)$

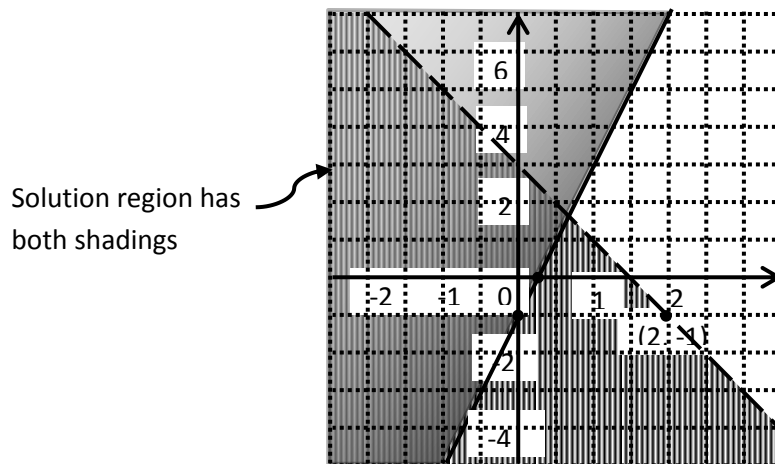
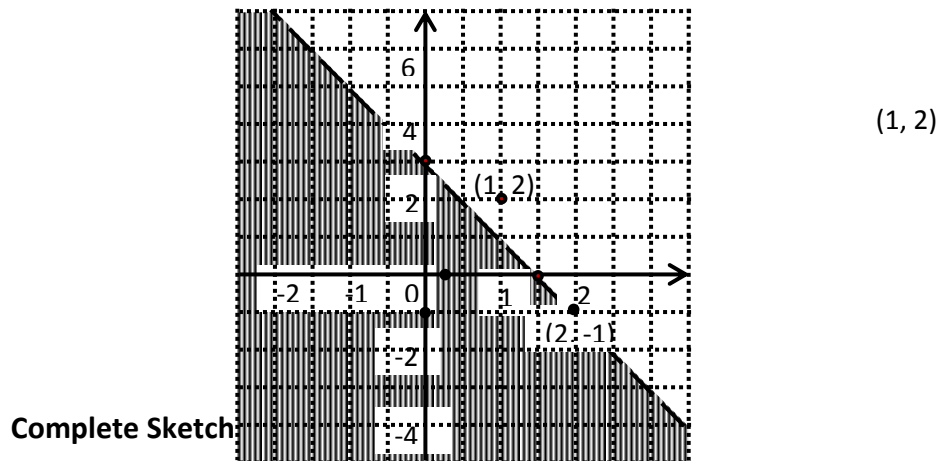


$$2 < -2.1 + 3$$

$$2 < -2 + 3$$

$$2 < 3 \text{ False}$$

Shade region that does not contain the point (1, 2)



The test points to be used can be any non-collinear point of the straight line. Use a point that is not on the straight line.

Example 7 Show the region that satisfies that inequalities $y \geq x^2 + 2x - 3$ and $y \leq -x + 3$.

Solution

$$\text{Intersection points } y \geq x^2 + 2x - 3$$

$$\text{When } y = 0, x^2 + 2x - 3 = 0$$

$$(x + 3)(x - 1) = 0$$

$$x = -3 \text{ or } 1$$

$$\text{When } x = 0, y = -3$$



Vertex $(-1, -4)$

$$\begin{aligned}x &= -b/2a \\&= -2/2(1) \\&= -2/2 \\&= -1\end{aligned}$$

$$\begin{aligned}\text{when } x &= -1 \\y &= (-1)^2 + 2(-1) - 3 \\&= 1 - 2 - 3 \\&= -4\end{aligned}$$

Therefore the TP or vertex is $(-1, -4)$

Your solution will be only one diagram. These three sketches for example 6 were given so you can go step by step to get at the complete sketch.

Region involving Quadratic Curve and a Straight Line

Follow the following **steps to sketch**.

Quadratic Curve

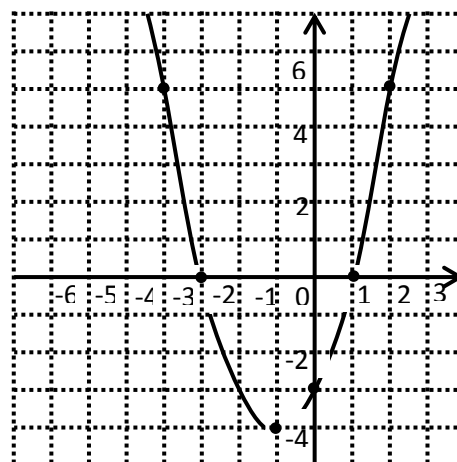
1. Where the curve cuts x-axis (roots)
2. Where the curve intersects y-axis ($y = c$)
3. The vertex
4. Whether to use a bold line or dash ($\leq$ and $\geq$ use bold, $<$ and $>$ use dash)
5. Shaded region

Linear Graph

1. Where the straight line intersects x-axis
2. Where the straight line intersects y-axis
6. Whether to rule a bold line or dash ($\leq$ and $\geq$ use bold, $<$ and $>$ use dash)
3. Shaded region

Graph of the curve $y = x^2 + 2x - 3$ (not $y \geq x^2 + 2x - 3$)

Test Point $(1, 3)$



**Region to shade**

$$y \geq x^2 + 2x - 3 \quad \text{Test Point } (1, 3)$$

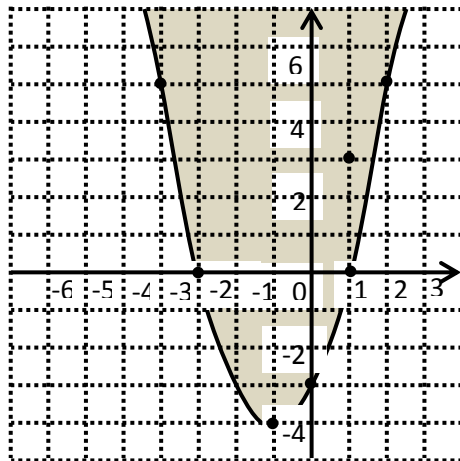
$$3 \geq 1^2 + 2(1) - 3$$

$$3 \geq 1 + 2 - 3$$

$$3 \geq 0 \quad \text{TRUE}$$

3 is not equal to zero, but is greater than zero.

Shade region that contains the point (1, 3)



Intersection points $y \leq -x + 3$

$$\text{When } y = 0, x = 4 \quad (0, 3)$$

$$\text{When } x = 0, y = 4 \quad (3, 0)$$

Region to shade

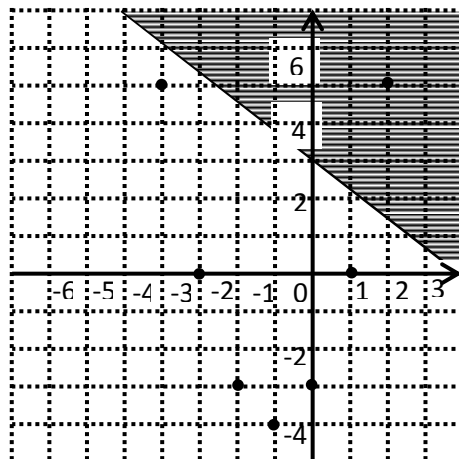
$$y \geq -x + 3 \quad \text{Test Point } (3, 2)$$

$$2 \geq -1(3) + 3$$

$$3 \geq -3 + 3$$

$$3 \geq 0 \quad \text{TRUE}$$

3 is greater than zero. Shade region containing the point (3, 2)

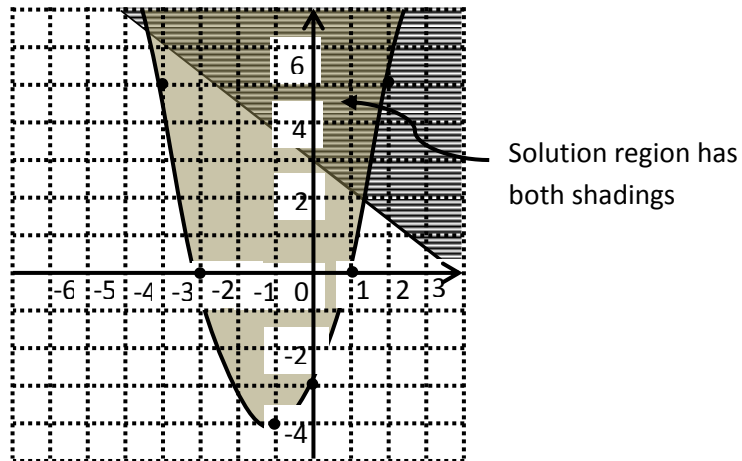


We now have the sketch of the second graph. We can proceed to placing graphs together.

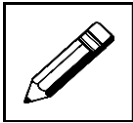


Your solutions will be only one diagram. These four sketches for example 7 respectively were given so you can go step by step to get at the complete sketch.

A Complete Sketch



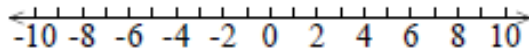
Though a grid is used here, in sketching we don't need to use a grid.

**STUDENT LEARNING ACTIVITY 11.1.4.4**

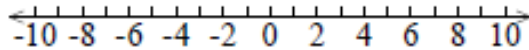
20 minutes

1. Plot the following inequalities on the number line.

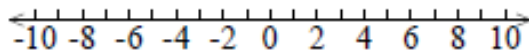
a) $x > 7$



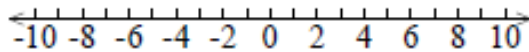
b) $x < 4$



c) $x \geq -1$

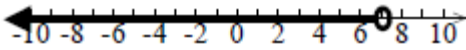


d) $x \leq 5$



2. Determine the inequalities shown below:

a)



$x < 7$

b)



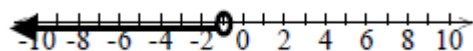
$x \geq -2$

c)



$x > -8$

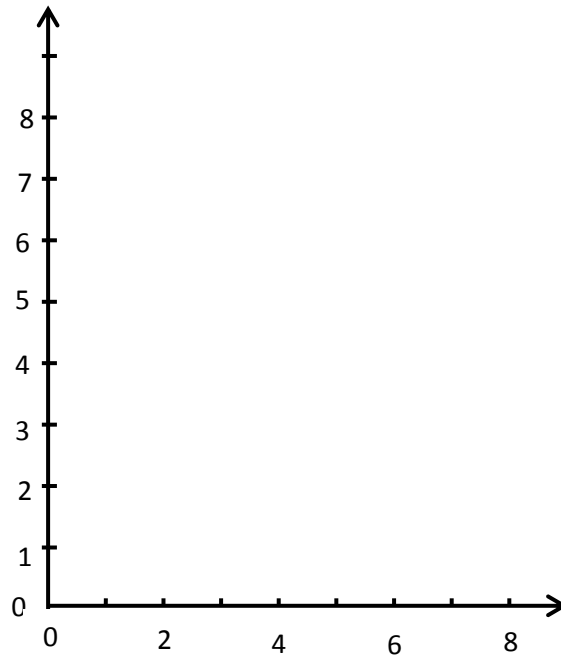
d)

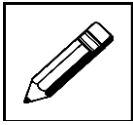


$x < -1$



3. Graph the inequality $x + 2y < 10$.



**SUMMATIVE ACTIVITY 11.1.4**

20 minutes

1) Factorise the following completely:

a) $x^2 + 7x = 12$

b) $x^2 + 3x - 10$

c) $x^2 - 6x + 9$

d) $8y + 12$

e) $3a + ab$

f) $m^2 + 8m + 12$

g) $y^2 - 64$

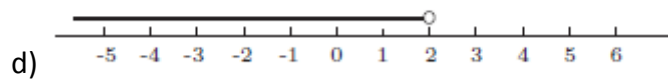
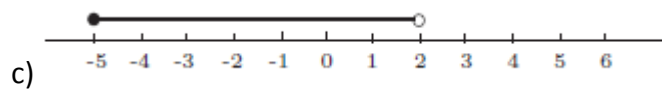
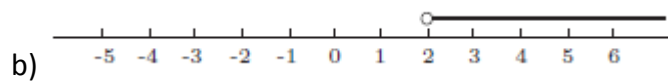
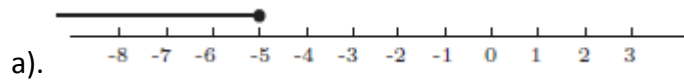
2) Simplify the following.

a) $\frac{5x + 8x}{x}$

b) $\frac{x^2 + 5x + 6}{x + 3}$



3) Write the inequality illustrated in the diagrams below:



4) Draw the following on the number line.

a) $x > -4$

b) $x < 2$



c) $X \geq 3$

d) $4 \leq x \leq 8$

e) $-6 \leq x \leq 2$

5) Find the solutions of the following:

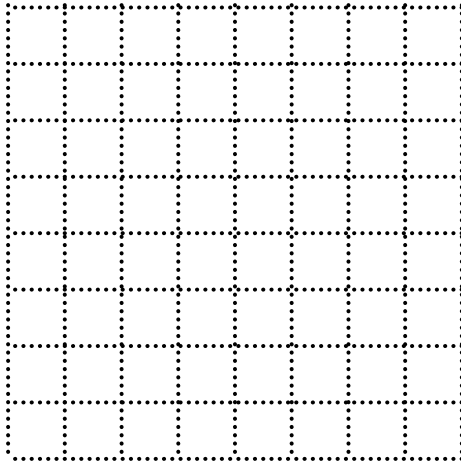
a) $(x - 2)(x + 3) \geq 0$

b) $x(x - 5) > 0$

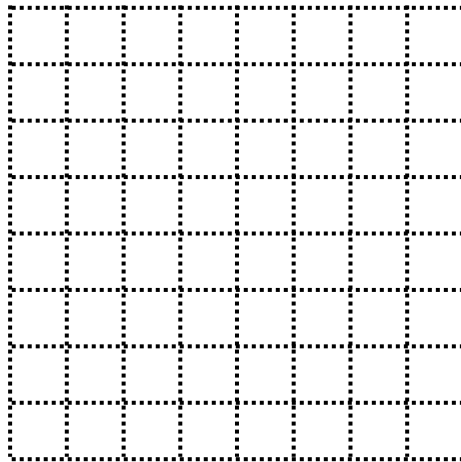


6) Sketch the graph of regions:

a) $Y \leq x^2 + x - 12$



b) $Y > 2x^2 - 2x - 4$





SUMMARY

This summary outlines the key ideas and concepts to be remembered.

- **Real Numbers** are those numbers that can be plotted on a number line. It means that these numbers consist of zero, all negative and all positive numbers including the numbers in between them.
- A **rational number** is any number that can be expressed as a ratio (or quotient) of two integers or simply expressed as a fraction. The set of rational numbers includes both integers and fractions.
- An irrational number on the contrary cannot be expressed as a fraction which yields to non-terminating and non-repeating decimals.
- **Properties of Real Numbers**
Commutative Properties: For any real numbers m and k , $m+k = k+m$ and $mk = km$
- **Associative Properties:** For any real numbers m , k , and v , $(m+k) + v = m + (k+v)$ and $(mk)v = m(kv)$
- **The Distributive Property:** For any real numbers a , n and v : $a(n+v) = an + av$ and $a(n-v) = an - av$
- **Identity Properties:** For any real number m : $m * 1 = 1 * m = m$ and $m + 0 = 0 + m = m$
- **Inverse property:** For every non zero real number m also has a multiplicative inverse or reciprocal, written $1/m$, such that $m * 1/m = 1$.
- **Factors** are the numbers or variables (letters) we multiply to get a number(product).
- **Multiples** are what we get after multiplying the number by any integer.
- Fraction is a numerical quantity that is not a whole number. Fraction is part of a whole.
- **Decimals** is a way of relating to or denoting a system of numbers and arithmetic based on the number ten, tenth parts, and powers of ten.
- The term significant figures refer to the number of important single digits (0 through 9 inclusive) in the coefficient of an expression in scientific notation.
- The difference between the actual sum and the estimated sum is called as the rounding error or **estimation error**.
- Laws of Indices:
 - Law 1: To multiply powers with the same base, add the indices. $b^m b^n = b^{m+n}$
 - Law 2: To raise a power to a power, multiply the indices. $(b^m)^n = b^{mn}$
 - Law 3: To get the power of a product, distribute and multiply the indices. $(ab)^m = a^m b^m$
 - Law 4: To divide powers with the same base, subtract the indices. $\frac{b^m}{b^n} = b^{m-n}$
where $m > n$
 - Law 5: To get the power of a quotient just find the quotient of the powers.
 $\frac{a}{b} = \frac{a^m}{b^m}$ where $b \neq 0$
 - Law 6: Any number or quantity raised to zero is equal to 1 $b^0 = 1$



- Law 7: Any number or quantity raised to a negative power is equal to its positive reciprocal. $b^{-m} = \frac{1}{b^m}$
- **Laws of Surds**
 - Law 1: $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
 - Law 2: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
 - Law 3: $(\sqrt{a})^2 = a$
- **Rationalizing the denominator** is a way of removing or eliminating surds in the denominator.
- To convert **from a larger unit** to a smaller unit, **multiply the amount of the larger unit by the conversion factor**.
- To convert **from a smaller unit** to a larger unit, **divide the amount of the smaller unit by the conversion factor**.
- A **perimeter** is a path that surrounds a two-dimensional shape. The word comes from the Greek peri (around) and meter (measure).
- Surface area is the total area of the surface of a three-dimensional object.

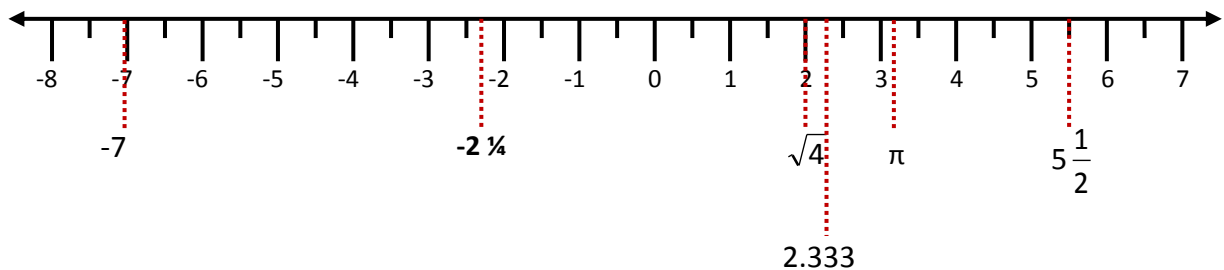


Answers to Students Learning Activities

STUDENT LEARNING ACTIVITY 11.1.1.1

- 1) a) rational b) irrational c) rational d) rational
- 2) a) real b) real c) real d) imaginary
 e) imaginary f) real g) real h) imaginary
 i) real j) imaginary k) real l) real
 m) real n) real

3)



- 4) a) Commutative Property of Addition b) Identity Property of Multiplication
 c) Inverse Property of Addition d) Distributive property
 e) Identity Property for Addition f) Associative Property

- 5) (a) $-9, \frac{1}{9}$ (b) $-2\sqrt[3]{6}, \frac{1}{2\sqrt[3]{6}}$ (c) $7, \frac{1}{7}$
 (d) $\sqrt{8}, -\frac{1}{\sqrt{8}}$ (e) $-\frac{6}{5}, \frac{5}{6}$ (f) $-3\frac{2}{7}, \frac{7}{23}$

6)

Vulgar	Decimal	Percentage
$\frac{9}{20}$	0.45	45%
$\frac{31}{50}$	0.62	62%
$\frac{3}{8}$	0.375	37.5%
$1\frac{1}{16}$	1.00625	100.625%
$\frac{21}{20}$	1.05	105%



STUDENT LEARNING ACTIVITY 11.1.1.2

1)

- a) 1,3,7,21
- b) 1,2,4,7,14,28
- c) 1,2,3,6,7,14,21,42
- d) 1,5,11,55
- e) 1,3,5,11,25,75

2) First ten multiples.

- a) $25 = 25, 50, 75, 100, 125, 150, 175, 200, 225, 250$
- b) $14 = 14, 28, 42, 56, 70, 84, 98, 112, 126, 150$
- c) $31 = 31, 62, 93, 124, 155, 186, 217, 248, 279, 310$

3)

x	-3	-2	-1	0	1	2	3
4	-12	-8	-4	0	4	8	12
6	-18	-12	-6	0	6	12	18
8	-24	-16	-8	0	8	16	24
10	-30	-20	-10	0	10	20	30

4) First five multiples.

- a) $7 = 7, 14, 21, 28, 35$
- b) $8 = 8, 16, 24, 32, 40$
- c) $13 = 13, 26, 39, 52, 65$
- d) $17 = 17, 34, 51, 68, 85$
- e) $20 = 20, 40, 60, 80, 100$

5) Product of Primes

- a) $130 = 2 \times 5 \times 13$
- b) $70 = 2 \times 5 \times 7$
- c) $200 = 2^3 \times 5^2$

6) Decimal to Binary

- a) $8_{10} = 1000_2$
- b) $12_{10} = 1100_2$
- c) $23_{10} = 10111_2$
- d) $37_{10} = 100101_2$
- e) $50_{10} = 110010_2$

7) Binary to Decimal

- a) $101_2 = 5_{10}$
- b) $1011_2 = 11_{10}$
- c) $11_2 = 3_{10}$
- d) $10_2 = 2_{10}$
- e) $110_2 = 6_{10}$



STUDENT LEARNING ACTIVITY 11.1.1.3

1)

a) $\frac{2}{3}$ is equivalent to $\frac{3}{2}, \frac{4}{6}, \frac{6}{12}, \frac{10}{15}, \frac{16}{24}$

b) $\frac{3}{4}$ is equivalent to $\frac{6}{8}, \frac{9}{12}, \frac{12}{15}, \frac{15}{20}, \frac{18}{24}$

2).

- a) 0.008
- b) 12.5
- c) 0.24
- d) 0.45
- e) 0.00237
- f) 0.72

STUDENT LEARNING ACTIVITY 11.1.1.4

2.

Given	Number of Significant Digits	List of Digits
a) 0.00002530000	3	2, 5, 3
b) 2.15×10^5	3	2, 1, 5
c) 120 000 000	2	1, 2
d) 12.00045	7	1, 2, 0, 0, 0, 4, 5
e) 15.23000000	4	1, 5, 2, 3
f) 2.05×10^8	3	2, 0, 5
g) 3.4×10^{-3}	2	3, 4
h) 2.000001	7	2, 0, 0, 0, 0, 0, 1
i) 3 000 000 000	1	3
j) 0.0025×10^{-4}	2	2, 5

3.

Number	Number of Decimal places	Number of Significant Figures
a) 0.0253	0.03 2dp	0.25 2sf
b) 2.0652	2.072 dp	2.07 3sf
c) 120	120.0 1dp	120 2sf
d) 12.045	12.045 3dp	12.0 3sf
e) 1.523	1.523 3dp	1.52 3sf



3.

- a) $2 \times 10^2 + 2 \times 10^{-1} = 200 + 0.2 = 200.2$ = 200
- b) $3.4 \times 10^2 + 4.5 \times 10^2 = 340 + 450 = 790$ = 790
- c) $9.32 \times 10^3 - 6.71 \times 10^2 = 9320 - 671 = 8649$ = 8 600
- d) $8.56 \times 10^{-1} - 9.7 \times 10^{-2} = 0.856 - 0.097 = 0.759$ = 0.76
- e) $4.1 \times 10^4 + 2.5 \times 10^3 = 41000 + 2500 = 43500$ = 44 000

4.

- a) $2.41 \times 10^2 \times 3.2 \times 10^3 = 2.41 \times 3.2 \times 10^3 \times 10^2 = 7.712 \times 10^{3+2} = 7.71 \times 10^5$
- b) $3.2 \times 10^3 \div 4.5 \times 10^2 = 3.2 \div 4.5 \times 10^3 \div 10^2 = 7.11111... \times 10^{3-2} = 7.11 \times 10^{-1}$
- c) $6.32 \times 10^3 \times 2.7 \times 10^{-2} = 6.32 \times 2.7 \times 10^3 \times 10^{-2} = 17.064 \times 10^{3-2} = 17.1 \times 10^5$
 $= 1.71 \times 10^6$
- d) $3.62 \times 10^{-1} \div 7.4 \times 10^{-2} = 3.62 \div 7.4 \times 10^{-1} \div 10^{-2} = 0.489 189 19... \times 10^{-1-(-2)}$
 $= 0.489 \times 10^1 = 4.89 \times 10^{-1} \times 10^1 = 4.89 \times 10^0$
- e) $1.4 \times 10^4 \times 5.3 \times 10^3 = 1.4 \times 5.3 \times 10^4 \times 10^3 = 7.42 \times 10^{4+3} = 7.32 \times 10^7$

STUDENT LEARNING ACTIVITY 11.1.1.5

1)

Given	Estimated value by rounding off to a whole number	Estimated value by rounding off to 1 significant figure
12.65	13	10
9.42	9	9
15.12	15	20
18.52	19	20
12.86	13	10

2)

- | | Given | Estimates | Estimated Answer |
|----|--------------------------|------------------|------------------|
| a) | $23.45 + 12.20 + 18.55$ | $=$ _____ | $=$ _____ |
| b) | $58.44 + 28.62 + 11.12$ | $= 58 + 29 + 11$ | $= 98$ |
| c) | $74.48 - 29.24$ | $= 74 - 29$ | $= 45$ |
| d) | $(12.82 - 6.24) + 15.67$ | $= 13 - 6 + 16$ | $= 23$ |

3)

- | | Given | Estimates | Estimated Answer |
|----|----------------------|------------------|------------------|
| a) | 12.55×89.20 | $= 10 \times 90$ | $= 900$ |
| b) | $52.48 \div 47.20$ | $= 50 \div 50$ | $= 1$ |
| c) | 18.42×64.08 | $= 20 \times 60$ | $= 1\ 200$ |
| d) | $121.42 \div 4.50$ | $= 100 \div 5$ | $= 20$ |

**SUMMATIVE TASK 11.1.1**

1. 6.11.
2. 7.12.
3. 8.13
4. 9.14.
5. 10.

STUDENT LEARNING ACTIVITY 11.1.2.1

1)

Index Form	Base	Exponent
4^8	4	8
7^{15}	7	15
$\left(\frac{2}{3}\right)^6$	$\left(\frac{2}{3}\right)$	6
x^5	x	5
2^n	2	n

- 2) (a) 4^5 (b) $(-3)^8$ (c) $16 m^5 n^3$

3)

	Expanded Form	Product
a) $(\sqrt{3})^6$	$= \sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3}$	$= 27$
b) $(-2)^8$	$= -2 \times -2 \times -2 \times -2 \times -2 \times -2 \times -2 \times -2$	$= 256$
c) $(2)^3 (\sqrt{3})^2$	$= 2 \times 2 \times 2 \times \sqrt{3} \times \sqrt{3}$	$= 24$
d) $(\sqrt[3]{8})(\sqrt{9})^3$	$= \sqrt[3]{8} \times \sqrt{9} \times \sqrt{9} \times \sqrt{9}$	$= 54$

4)

	Expanded Form	Product
a) 3^8	$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$	$= 6561$
b) $(-2)^6$	$= -2 \times -2 \times -2 \times -2 \times -2 \times -2$	$= 64$
c) $(2)^3 (3)^2$	$= 2 \times 2 \times 2 \times 3 \times 3$	$= 72$
d) $(\sqrt{3})^3 (\sqrt{2})^3$	$= \sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2}$	$= 6\sqrt{6}$

- 5) a) $\left(\frac{2}{5}\right)^4 = \frac{16}{625}$
- b) $\left(\frac{\sqrt[4]{81}}{3^2}\right)^{-1} = \left(\frac{3}{9}\right)^{-1} = \left(\frac{9}{3}\right) = 3$
- c) $\left(\frac{\sqrt[5]{3125}}{\sqrt[3]{64}}\right)^2 = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$



$$d) \left(\frac{\sqrt{6}}{\sqrt{3}} \right)^2 \times \left(\frac{\sqrt{4}}{4} \right) = \frac{6}{3} \times \frac{2}{4} = 1$$

STUDENT LEARNING ACTIVITY 11.1.2.2

1)

- | | | |
|----|-------------------|-------|
| a) | $8x^3$ | Law 3 |
| b) | 128 | Law 1 |
| c) | 6561 | Law 2 |
| d) | $9a^2b^2$ | Law 3 |
| e) | 2^{3x} | Law 2 |
| f) | x^9 | Law 1 |
| g) | $4096x^8$ | Law 3 |
| h) | $x^5y^{10}z^{10}$ | Law 1 |
| i) | 3^{4b} | Law 2 |
| j) | $3^kx^ky^k$ | Law 3 |

3) Simplify the following.

$$1. \left(\frac{1}{2x} \right)^{-2} = \left(\frac{2x}{1} \right)^2 = 4x^2$$

$$2. 2^{-\frac{1}{2}}\sqrt{2} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

$$3. (x - \sqrt{3})^2 = (x^2 - \sqrt{3}x - \sqrt{3}x + 3) = x^2 - 2\sqrt{3}x + 3$$

$$4. \left(\frac{2^{-2}}{3^{-3}} \right) \left(\frac{\sqrt{32}}{\sqrt{18}} \right)^{-1} = \left(\frac{3^3}{2^2} \right) \left(\frac{\sqrt{18}}{\sqrt{32}} \right) = \frac{27 \times 3\sqrt{2}}{4 \times 4\sqrt{2}} = \frac{81}{16}$$

$$5. \left(\frac{\sqrt{7}}{2\sqrt{3}} x \sqrt{5} \right)^2 = \left(\frac{\sqrt{35}}{2\sqrt{3}} \right)^2 = \frac{35}{4 \cdot 3} = \frac{35}{12}$$

$$6. x^{\frac{1}{3}} x^{-\frac{2}{3}} x = \sqrt[3]{x} \cdot \frac{1}{\sqrt[3]{x^2}} \cdot x = \frac{x}{\sqrt[3]{x}} = \sqrt[3]{x}$$

$$4. \left(\frac{3}{8} \right)^{-\frac{1}{2}} \left(\frac{8}{3} \right)^{-2} = \left(\frac{8}{3} \right)^{\frac{1}{2}} \left(\frac{3}{8} \right)^2 = \frac{2\sqrt{2}}{\sqrt{3}} \cdot \frac{9}{64} = \frac{9\sqrt{2}}{32\sqrt{3}}$$

$$5. 2^{1-2x} \cdot 4^{x-3} = 2^{1-2x} \cdot 2^{2(x-3)} = 2^{1-2x} \cdot 2^{2x-6} = 2^{-5} = \frac{1}{32}$$

$$6. 10^x \cdot 10^{\frac{1}{2}} = 10^{x+\frac{1}{2}} = 10^{2x+1}$$



$$7. \quad \left(\sqrt[k]{xy^2z^3} \right)^k = \left[(xy^2z^3)^{\frac{1}{k}} \right]^k = xy^2z^3$$

STUDENT LEARNING ACTIVITY 11.1.2.3

$$1) \quad a) \frac{1}{27} \quad b) 8 \quad c) \frac{4x^4y^6}{25z^8} \quad d) a^2b^7c \quad e) a^6$$

$$2) \quad a) \frac{xy^2}{z^3} \quad b) \frac{9}{2} \text{ or } 4\frac{1}{2} \quad c) 6 \quad d) \frac{117}{25} \text{ or } 4\frac{17}{25}$$

$$3) \quad a) \frac{4x^2}{9} \quad b) 16 \quad c) \frac{\sqrt[3]{2x^2y}}{3z} \quad d) a^2b^2c^2 \quad e) a$$

STUDENT LEARNING ACTIVITY 11.1.2.4

$$1) \quad 3 \quad 6. \quad \frac{1}{4} \quad 11. \quad \frac{1}{64}$$

$$2) \quad 3 \quad 7. \quad \frac{4}{x^5} \quad 12. \quad 5$$

$$3) \quad \frac{1}{9} \quad 8. \quad \frac{49}{4} \quad 13. \quad 16$$

$$4) \quad 2 \quad 9. \quad \frac{x^5}{y^5} \quad 14. \quad -1$$

$$5) \quad 3 \quad 10. \quad \frac{8}{x^3} \quad 15. \quad \frac{1}{225}$$

STUDENT LEARNING ACTIVITY 11.1.2.5

$$1. \quad x = 3 \quad 6. \quad x = 3$$

$$2. \quad x = -\frac{1}{2} \quad 7. \quad x = 1$$

$$3. \quad x = 5 \quad 8. \quad x = 4$$

$$4. \quad x = 3 \quad 9. \quad x = 5$$

$$5. \quad x = 4 \quad 10. \quad x = -5$$

SUMMATIVE TASK 11.1.2

$$1) \quad \text{Power, index or exponent} \quad 2) \quad x^5y^3z^2 \quad 3) \quad 2 \times 2 \times 2 \times x \times x \times x \times y \times y \times y$$

$$4) \quad \frac{1}{x^4y^2z^2} \quad 5) \quad x = 2 \quad 6) \quad \frac{3}{64}$$

$$7) \quad x = 3 \quad 8) \quad 8 - 7x \quad 9) \quad 4y^2$$



- 10) a) 4 b) $\frac{1}{6561}$ c) $\frac{z^8}{x^4y^6}$ d) $a^5b^{10}c^6$
- e) 1 f) 4 g) -6 h) $\frac{1}{8}$
- i) $\frac{1}{2}$ j) -2 k) $-\frac{1}{3}$ l) $\frac{3}{x^2}$
- m) $\frac{9}{x^2}$ n) x^8 o) $\frac{5}{2x}$

STUDENT LEARNING ACTIVITY 11.1.3.1

1) Fractional exponents to surds

Indices / Fractional Exponent Form	Base (Radicand)	Denominator of the fractional exponent (index)	Numerator of the fractional exponent (exponent of the radicand)	Surd / Radical Form	Surds read as...
a) $y^{\frac{1}{3}}$	y	3	1	$\sqrt[3]{y}$	Cube root of y
b) $2^{\frac{6}{7}}$	2	7	6	$\sqrt[7]{2^6}$	7 th root of 2 to the 6 th power
c) $9^{\frac{1}{5}}$	9	5	1	$\sqrt[5]{9}$	5 th root of 9

2) Surds to fractional exponents

Given	Surd / Radical Form	Radicand (Base)	Index (Denominator of the fractional exponent)	Exponent of the radicand (Numerator of the fractional exponent)	Indices / Fractional Exponent Form
"the cube root of x raised to y"	$\sqrt[3]{x^y}$	x	3	y	$x^{\frac{y}{3}}$
"the nth root of 3 cubed"	$\sqrt[n]{3^3}$	3	n	3	$3^{\frac{3}{n}}$
"the ninth root of 3x raised to 5"	$\sqrt[9]{(3x)^5}$	3x	9	5	$(3x)^{\frac{5}{9}}$

- 3) $xy^{\frac{1}{2}}$
- 4) $a^{\frac{3}{2}}b^3c^4$
- 5) $\sqrt[3]{w^{12}x^6y}$
- 6) 0.25 or $\frac{1}{4}$



STUDENT LEARNING ACTIVITY 11.1.3.2

1)

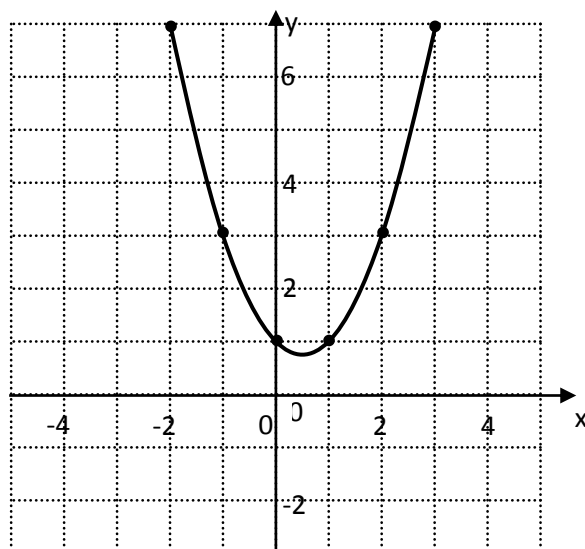
- a) $2\sqrt{10}$ Law 1
- b) $2x$ Law 3
- c) $2\sqrt[3]{5}$ Law 1
- d) $\frac{2\sqrt{3}}{5}$ Law 2
- e) $17x^2$ Law 3
- f) $\frac{1}{3}$ Law 2
- g) $10\sqrt{2}$ Law 1
- h) $\frac{3}{7}$ Law 2
- i) 51 Law 3
- j) $9\sqrt[3]{10}$ Law 1

2)

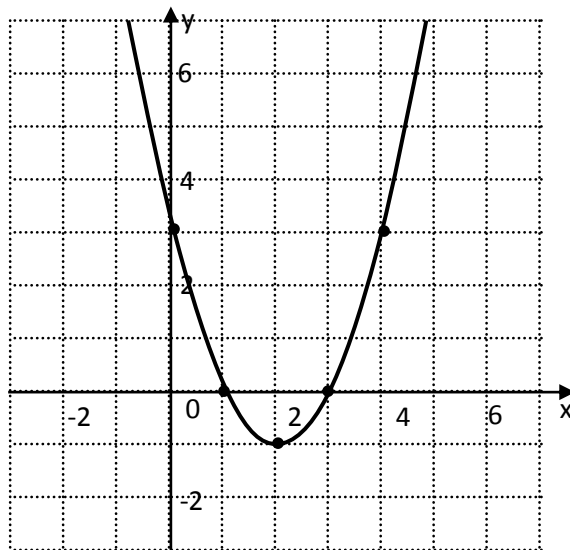
- a) ± 8
- b) $\pm 5i$
- c) $-3 \pm 2\sqrt{5}$
- d) -6

3)

a) $y = x^2 - x + 1$



b) $y = x^2 - 4x + 3$



STUDENT LEARNING ACTIVITY 11.1.3

Perform the indicated operation(s) involving surds.

$$1) -5\sqrt{5} + 12\sqrt{5} = 7\sqrt{5}$$

$$2) \sqrt{40} \times \sqrt{8} = 8\sqrt{5}$$

$$3) \sqrt{5} (3\sqrt{5} - 2\sqrt{5}) = 5$$

$$4) \frac{\sqrt{16}}{\sqrt{6}} = \frac{2\sqrt{6}}{3}$$

$$5) \sqrt{3} (4 + \sqrt{25}) = 9\sqrt{3}$$

$$6) -6\sqrt{20} + 2\sqrt{45} + 7\sqrt{5} = \sqrt{5}$$

$$7) -4\sqrt{20} + 7\sqrt{5} + 3\sqrt{45} = 8\sqrt{5}$$

$$8) 13\sqrt{3} + 18\sqrt{81} = 13\sqrt{3} + 162$$

$$9) \frac{2}{\sqrt{8}} = \frac{\sqrt{2}}{2}$$

$$10) -\sqrt{2} (\sqrt{3} + \sqrt{27}) = -4\sqrt{6}$$

$$11) (-8 - \sqrt{2}) (-8 + \sqrt{2}) = 62$$

$$12) 2\sqrt{24} \div \sqrt{144} = \frac{\sqrt{6}}{3}$$

$$13) \frac{\sqrt{8}}{\sqrt{5}} = \frac{2\sqrt{10}}{5}$$

$$14) (\sqrt{2} + 3)(\sqrt{2} - 3) = -7$$



15)

Expression	Conjugate	Product
a) $-7 + \sqrt{5}$	$-7 - \sqrt{5}$	44
b) $\sqrt{2} + \sqrt{3}$	$\sqrt{2} - \sqrt{3}$	-1
c) $\sqrt{2} - 5$	$\sqrt{2} + 5$	-5
d) $-\sqrt{6} + 4$	$-\sqrt{6} - 4$	-10
e) $-2y^2 + \sqrt{y^4}$	$-2y^2 - \sqrt{y^4}$	$3y^4$

16) $2 - \sqrt{3}$

17) Simplify: $\frac{\sqrt{32}}{4 - \sqrt{3}} = \frac{\sqrt{32}(4 + \sqrt{3})}{16 - 3} = \frac{16\sqrt{2} + 4\sqrt{6}}{13}$

18) $(\sqrt{5} - 6)(3 + \sqrt{3}) = 3\sqrt{5} + \sqrt{15} - 18 - 6\sqrt{3}$

19) $(\sqrt{5} - \sqrt{3})(\sqrt{2} + \sqrt{3}) = \sqrt{10} + \sqrt{15} - \sqrt{6} - 3$

20) $3 - \sqrt{5} + \sqrt{180} = 3 + 5\sqrt{5}$

STUDENT LEARNING ACTIVITY 11.1.4.3.1

1)

(a) $x^2 + 12x + 27$

(b) $x^2 - 3x - 18$

(c) $x^2 + 2x - 24$

(d) $x^2 - 13x + 22$

(e) $2x^2 + 9x - 18$

(f) $3x^2 - 5x - 12$

(g) $2 - 3x - 20x^2$

(h) $30 + 7x - 2x^2$

2)

a) $3(x + 2)(x - 2) = 0$

a) $x(x + 3) = 0$

b) $x(x - 8) = 0$

c) $(x - 10)(x + 10) = 0$

3)

a) $3(x - 2)(x - 2) = 0$

$(a - b)(a - b) = a^2 - 2ab + b^2$



$$\begin{array}{ll} \text{b) } (x+7)(x+7) = 0 & (a+b)(a+b) = a^2 + 2ab + b^2 \\ \text{c) } (x-10)(x+10) = 0 & (a+b)(a-b) = a^2 - b^2 \\ \text{d) } (x-10)(x+8) = 0 & (x+a)(x+b) = x^2 + (a+b)x + ab \end{array}$$

4)

$$\begin{array}{ll} \text{a) } 6(x-1)(x-1) = 0 & \text{therefore } x = 1 \text{ or } x = 1 \\ \text{b) } (x+11)(x-8) = 0 & \text{therefore } x = -11 \text{ or } x = 8 \\ \text{c) } x(4x-1) = 0 & \text{therefore } x = 0 \text{ or } x = 1/4 \\ \text{d) } (x-10)(x-10) = 0 & \text{therefore } x = 10 \text{ or } x = 10 \\ \text{e) } (x-5)(x+5) = 0 & \text{therefore } x = 5 \text{ or } x = -5 \\ \text{f) } 3x(2x-1) = 0 & \text{therefore } x = 0 \text{ or } x = 1/2 \\ \text{g) } (x-3)(x-3) = 0 & \text{therefore } x = 3 \text{ or } x = 3 \\ \text{h) } (x+9)(x+9) = 0 & \text{therefore } x = -9 \text{ or } x = -9 \\ \text{i) } 2(x+3)(x+3) = 0 & \text{therefore } x = -3 \text{ or } x = -3 \end{array}$$

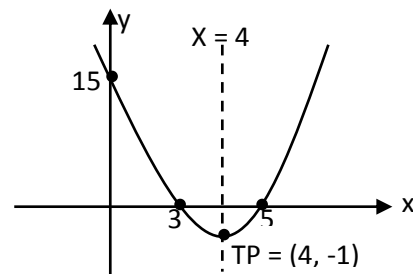
5)

- a) $D > 0$ therefore has two distinct real roots
- b) $D > 0$ therefore has two distinct real roots
- c) $D < 0$ therefore has no real roots
- d) $D = 0$ therefore has two equal real roots

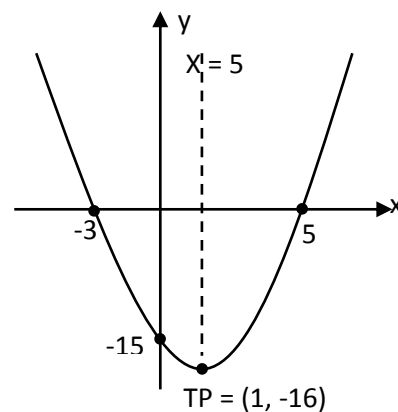
STUDENT LEARNING ACTIVITY 11.1.4.3.1 (b)

1) Sketch

$$\begin{array}{ll} \text{a) } y = x^2 - 8x + 15 & c = 15 \\ (x-3)(x-5) = 0 & \text{axis } x = 8/2 = 4 \\ x = 3 \text{ or } 5 & \text{Vertex} = (4, -1) \end{array}$$



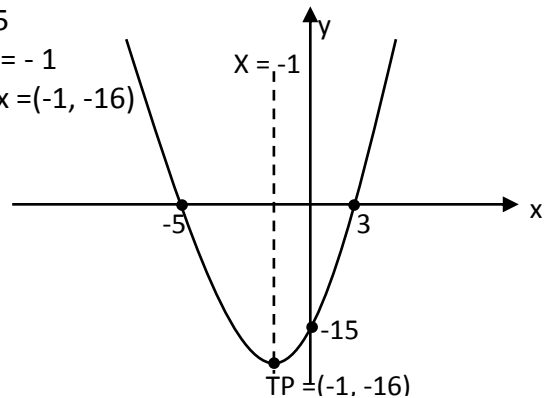
$$\begin{array}{ll} \text{b) } y = x^2 - 2x - 15 & c = -15 \\ (x+3)(x-5) = 0 & \text{axis } x = 2/2 = 1 \\ x = -3 \text{ or } 5 & \text{Vertex} = (1, -16) \end{array}$$





c) $y = x^2 + 2x - 15$
 $(x+5)(x-3) = 0$
 $x = -5 \text{ or } 3$

$c = -15$
axis $x = -1$
Vertex $= (-1, -16)$



2. $y = a(x - h)^2 + k$

a) $y = 2x^2 - 10x + 30$

$$\begin{aligned} & (2x^2 - 10x) + 30 \\ & 2(x^2 - 5x) + 15 \\ & 2\left(x^2 - (5/2)x + (5/2)^2\right) + 15 - (25/4) \\ & 2\left(x - 5/2\right)^2 + 60/4 - 25/4 \\ & 2\left(x - 5/2\right)^2 + 35/4 \\ & 2\left(x - 5/2\right)^2 + 8\frac{3}{4} \end{aligned}$$

b) $y = 4x^2 + 5x - 6$

$$\begin{aligned} & 4x^2 + 5x - 6 \\ & 4\left(x^2 + (5/4)x\right) - 6/4 \\ & 4\left(x^2 + (5/8)x + 25/64\right) - 3/2 - 25/64 \\ & 4\left(x + 5/8\right)^2 - (96/64 + 25/64) \\ & 4\left(x + 5/8\right)^2 - 121/64 \end{aligned}$$

c) $y = x^2 + 10x - 11$

$$\begin{aligned} & x^2 + 10x - 11 \\ & (x^2 + 10x) - 11 \\ & (x^2 + 5x + 25) - 11 - 25 \\ & (x + 5)^2 - 36 \end{aligned}$$



3. a) $y = 2x^2 + 3x - 5$ b) $y = 3x^2 + 4x - 4$ c) $2 - 3x - 2x^2 = 0$ d) $-6x^2 + x + 2 = 0$

$$\begin{aligned}
 x &= \frac{-3}{2 \cdot 2} \pm \frac{\sqrt{3^2 - 4 \cdot 2 \cdot -5}}{2 \cdot 2} & x &= \frac{-4}{2 \cdot 3} \pm \frac{\sqrt{4^2 - 4 \cdot 3 \cdot -4}}{2 \cdot 3} & x &= \frac{-3}{2 \cdot -2} \pm \frac{\sqrt{-3^2 - 4 \cdot -2 \cdot 2}}{2 \cdot -2} & x &= \frac{-1}{2 \cdot -6} \pm \frac{\sqrt{1^2 - 4 \cdot -6 \cdot 2}}{2 \cdot -6} \\
 &= -\frac{3}{4} \pm \frac{\sqrt{9+40}}{4} & &= -\frac{4}{6} \pm \frac{\sqrt{16+48}}{6} & &= -\frac{3}{4} \pm \frac{\sqrt{9+16}}{-4} & &= \frac{1}{12} \pm \frac{\sqrt{1+48}}{12} \\
 &= -\frac{3}{4} \pm \frac{\sqrt{49}}{4} & &= -\frac{4}{6} \pm \frac{\sqrt{64}}{6} & &= -\frac{3}{4} \pm \frac{\sqrt{25}}{-4} & &= \frac{1}{12} \pm \frac{\sqrt{49}}{12} \\
 &= -\frac{3}{4} \pm \frac{7}{4} & &= -\frac{4}{6} \pm \frac{8}{6} & &= -\frac{3}{4} \pm \frac{5}{-4} & &= \frac{1}{12} \pm \frac{7}{12} \\
 &= -\frac{10}{4} \text{ or } \frac{4}{4} & &= -\frac{12}{6} \text{ or } \frac{4}{6} & &= \frac{2}{4} \text{ or } -\frac{8}{6} & &= \frac{8}{12} \text{ or } -\frac{6}{12} \\
 &= -\frac{5}{2} \text{ or } 1 & &= -2 \text{ or } \frac{2}{3} & &= \frac{1}{2} \text{ or } -\frac{4}{3} & &= \frac{2}{3} \text{ or } -\frac{1}{2}
 \end{aligned}$$

STUDENT LEARNING ACTIVITY 11.1.4.3.1 (c)

1)

- a) $(x-2)(x-4) = 0$, $x = 2$ and $x = 4$
 b) $(x+1)(x+9) = 0$, $x = -1$ and $x = -9$
 c) $(x-3)(x-3) = 0$, $x = 3$ and $x = 3$
 d) $(x-2)(x+5) = 0$, $x = 2$ and $x = -5$

4. completing –the-square method.

a) $x^2 - 7x + 12 = 0$

$$\begin{aligned}
 x^2 - 7x &= -12 \\
 x^2 - 7x + (7/2)^2 &= -12 + 49/4 \\
 (x - 7/2)^2 &= 1/4 \\
 (x - 7/2) &= \pm 1/2 \\
 x &= 7/2 \pm 1/2 \\
 x &= 4 \text{ or } 3
 \end{aligned}$$

b) $x^2 + 2x - 8 = 0$

$$\begin{aligned}
 x^2 + 2x &= 8 \\
 x^2 + 2x + (1)^2 &= 8 + 1 \\
 (x + 1)^2 &= 9 \\
 x + 1 &= \pm 3 \\
 x &= -1 \pm 3 \\
 x &= -4 \text{ or } 2
 \end{aligned}$$



c) $2x^2 + 5x - 3 = 0$

$$\begin{aligned} 2x^2 + 5x &= 3 \\ x^2 + (5/2)x &= 3/2 \\ (x + (5/4)x + (5/4)^2) &= 3/2 + 25/16 \\ (x + 5/4)^2 &= 49/16 \\ x + 5/4 &= \pm 7/4 \\ x &= -5/4 \pm 7/4 \end{aligned}$$

$$x = -3 \text{ or } \frac{1}{2}$$

d) $3x^2 + 10x + 8 = 0$

$$\begin{aligned} 3x^2 + 10x &= -8 \\ x^2 + (10/3)x &= -8/3 \\ x^2 + (10/6)x + (10/6)^2 &= -8/3 + 100/36 \\ (x + 10/6)^2 &= 4/36 \\ (x + 10/6) &= \pm 2/6 \\ x &= 10/6 \pm 2/6 \end{aligned}$$

$$x = 2 \text{ or } 4/3$$

3) curve $y = x^2 - 8x + 15$

a) axis of symmetry at $x = -b/2a$ vertex at $x = 4$
 $= 8/2 \times 1$ $y = 4^2 - 8 \times 4 + 15$
 $= 4$ $= 16 - 32 + 15$
 $= -1$

Vertex (4, -1)

b) vertex of the curve $y = 2x^2 + 5x + 2$
 $(2x^2 + 5x) + 2$
 $2(x^2 + 5/2 x) + 2$
 $2(x^2 + 5/2 x + (1/4)^2) + 2 - 1/16$
 $2(x + 1/4)^2 + 1 \frac{15}{16}$

$$\text{Vertex (h, k) = } (-\frac{1}{4}, 1 \frac{15}{16})$$

STUDENT LEARNING ACTIVITY 11.1.4.3

1) Use the discriminant to check if the following have real roots.

a) $x^2 - 8x + 16 = 0$	$\Delta = (-8)^2 - 4.1.16 = 64 - 64 = 0$	has two equal real roots
b) $x^2 - 13x + 30 = 0$	$\Delta = (-13)^2 - 4.1.30 = 169 - 120 = 49 > 0$	has two distinct real roots
c) $2x^2 - 5x - 3 = 0$	$\Delta = (-5)^2 - 4.2.-3 = 25 - 24 = 1 > 0$	has two distinct real roots
d) $x^2 + 2x + 5 = 0$	$\Delta = (2)^2 - 4.1.5 = 4 - 20 = -16 < 0$	has no real roots
e) $2x^2 - 12x + 18 = 0$	$\Delta = (-12)^2 - 4.2.18 = 144 - 144 = 0$	has two equal real roots



2) Find the roots of the following using any method.

- a) $x^2 = 64$, $x = \pm 8$, $x = -8$ or 8
 b) $3x^2 + 75 = 0$, $3x^2 = -75$, $x^2 = -25$, $x = \pm \sqrt{-25}$, $x = \sqrt{-25}$ or $-\sqrt{-25}$
 c) $(x+3)^2 = 20$, $x + 3 = \pm 2\sqrt{5}$, $x = -3 \pm 2\sqrt{5}$, $x = -3 - 2\sqrt{5}$ or $-3 + 2\sqrt{5}$
 d) $x^2 + 12x + 36 = 0$, $(x + 6)(x + 6) = 0$, $x = -6$ or -6

3) Sketch the following quadratic functions.

a) $y = x^2 - x - 6$

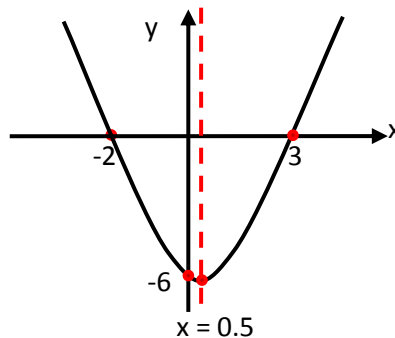
$$(x + 2)(x - 3) = 0$$

Roots: $x = -2$, $x = 3$

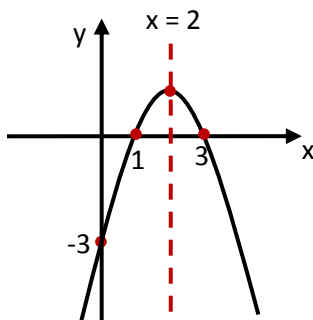
Y – intercept $c = -6$

$$1(x - 0.5)^2 - 6.25 = 0$$

Axis of symmetry $x = 0.5$
 Vertex $(0.5, -6.25)$
 MIN



b) $y = -x^2 + 4x - 3$



$$-1(x^2 - 4x + 3) = 0$$

$$-1(x - 1)(x - 3) = 0$$

Roots: $x = 1$, $x = 3$

Y – intercept $c = -3$

$$1(x - 2)^2 + 1 = 0$$

Axis of symmetry $x = 2$
 Vertex $(2, 1)$
 MAX

4) Complete- the -square

a) $2x^2 + 11x + 12 = 0$

$$2x^2 + 11x = -12$$

$$x^2 + (11/2)x = -6$$

$$x^2 + (11/4)x + 121/16 = -6 + 7^9/16$$

$$(x + 11/4)^2 = 25/16$$

$$(x + 11/4) = \pm 5/4 \quad \text{[take square root]}$$

$$x = 11/4 \pm 5/4$$

$$x = 4 \text{ or } 3/2$$



b) $y = x^2 - 4x - 12$

$$\begin{aligned}x^2 - 4x &= 12 \\x^2 - 2x + 2^2 &= 12 + 4 \\(x - 2)^2 &= 16 \\x - 2 &= \pm 4 \\x &= 2 \pm 4\end{aligned}$$

[take square root]

$x = -2 \text{ or } 6$

c) $2x^2 + 5x - 12 = 0$

$$\begin{aligned}2x^2 + 5x &= 12 \\x^2 + \frac{5}{2}x &= 6 \\x^2 + \frac{5}{4}x + \frac{25}{16} &= 6 + \frac{25}{16} \\(x + \frac{5}{4})^2 &= \frac{121}{16} \\x + \frac{5}{4} &= \pm \frac{11}{4} \\x &= -\frac{5}{4} \pm \frac{11}{4}\end{aligned}$$

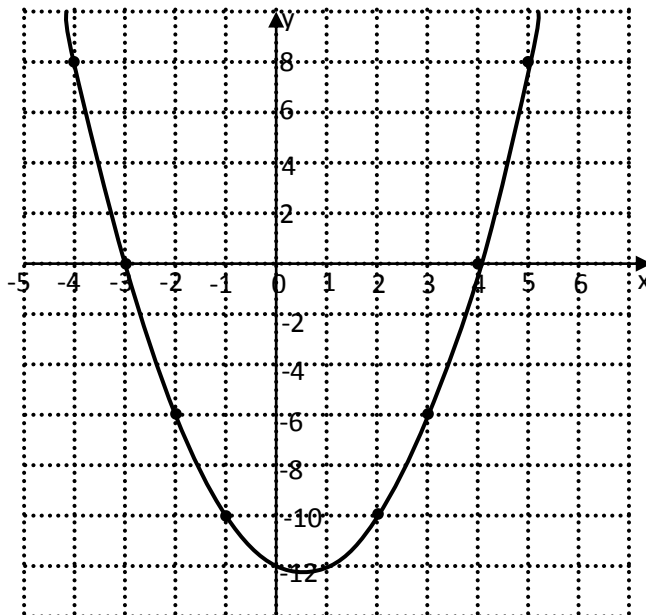
[take square root]

$x = -4 \text{ or } \frac{3}{2}$

5) Graph the following quadratic functions.

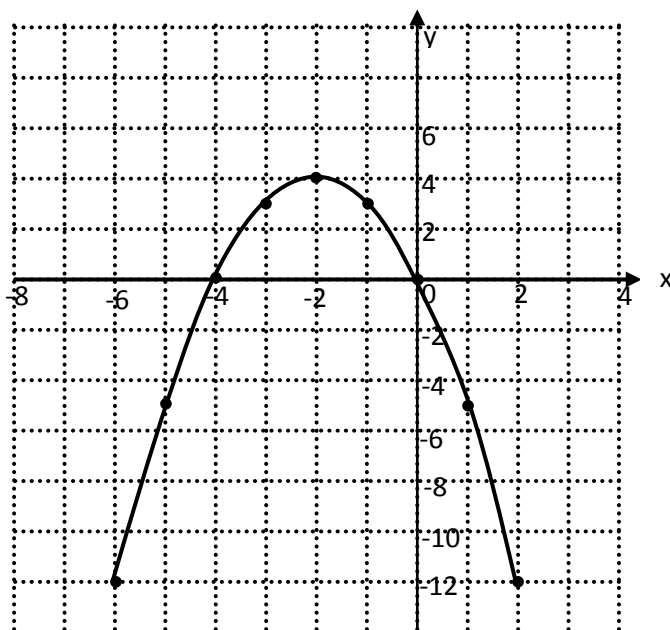
a) $y = x^2 - x - 12$

X	-4	-3	-2	-1	0	1	2	3	4	5
y	8	0	-6	-10	-12	-12	-10	-6	0	8



b) $y = -x^2 - 4x$

X	-6	-5	-4	-3	-2	-1	0	1	2
y	-12	-5	0	3	4	3	0	-5	-12

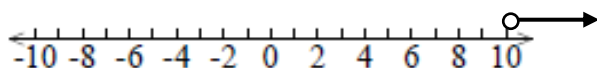


- 6) axis of symmetry and vertex, $y = x^2 - 8x + 15$
 $x^2 - 8x = 15$
 $x^2 - (8/2)x + 4^2 + 15 - 16$ axis of symmetry $x = 4$
 $(x - 4)^2 - 1$ Vertex = $(4, -1)$
- 7) vertex, $y = 2x^2 + 5x + 2$
 $(2x^2 + 5x) + 2$
 $2(x^2 + 5/2 x) + 2$
 $2(x^2 + 5/4 x + 25/16) + 2 - 1^{9/16}$
 $2(x + 5/4)^2 - 7/16$ Vertex = $(-5/4, -7/16)$

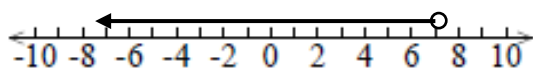
STUDENT LEARNING ACTIVITY 11.1.4.4

1)

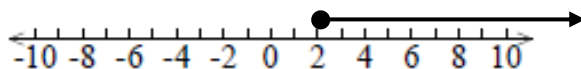
a) $x > 7$



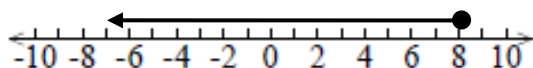
b) $x < 4$



c) $x \geq -1$



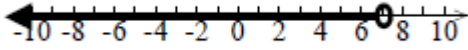
d) $x \leq 5$





2) Determine the inequalities shown below:

a)



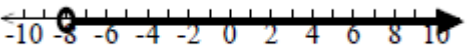
$$x < 7$$

b)



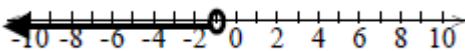
$$x \geq -2$$

c)



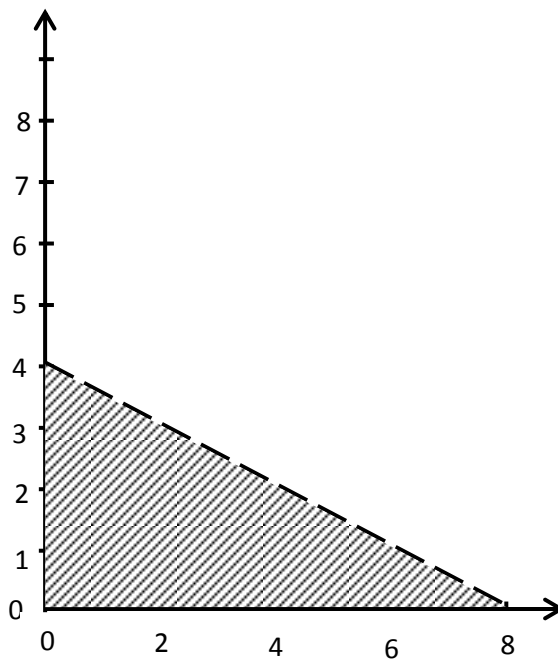
$$x > -8$$

d)



$$x \leq -1$$

3) Graph the inequality $x + 2y < 8$.



$$2y = -x + 8$$

$$X = 0, y = 4 \quad (0, 4)$$

$$Y = 0, x = 8 \quad (8, 0)$$

SUMMATIVE TASK 11.1.4.

1) Factorise the following completely:

a) $x^2 + 7x = -12$ $(x + 4)(x + 3) = 0$

b) $x^2 + 3x - 10$ $(x + 5)(x - 2) = 0$

c) $x^2 - 6x + 9$ $(x - 3)(x - 3) = 0$

d) $8y + 12$ $4(2y + 3)$

e) $3a + ab$ $a(3 + b)$

f) $m^2 + 8m + 12$ $(m + 2)(m + 6)$

g) $y^2 - 64$ $(y - 8)(y + 8)$

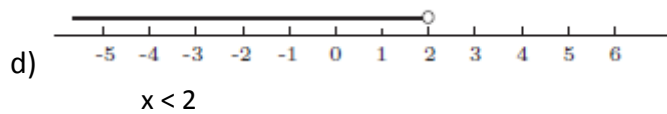
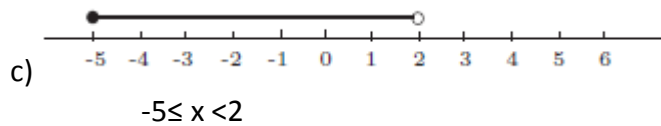
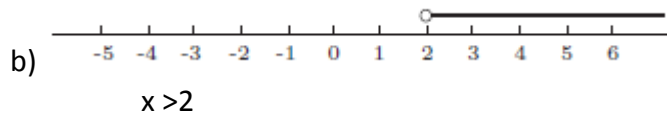
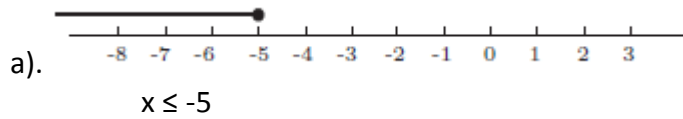


2) Simplify the following.

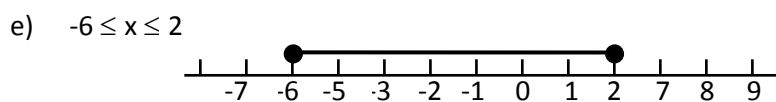
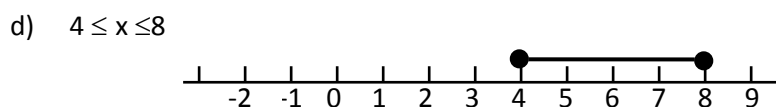
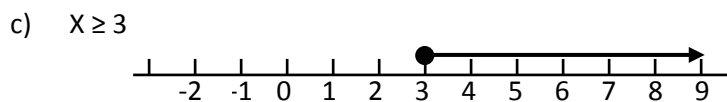
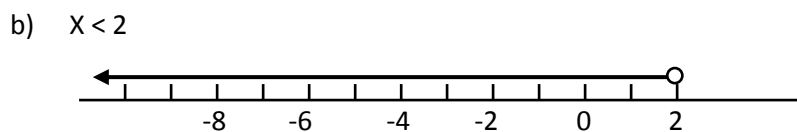
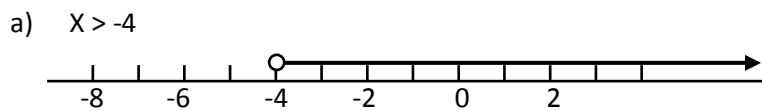
a) $\frac{5x + 8x}{x} = \frac{x(5 + 8)}{x} = 5 + 8 = 13$

b) $\frac{x^2 + 5x + 6}{x + 3} = \frac{(x + 2)(x + 3)}{x + 3} = x + 2$

3) Write the inequality illustrated in the diagrams below:



4) Draw the following on the number line.





5)

$$\text{a) } (x-2)(x+3) \geq 0 \quad \text{when} \quad \begin{array}{l} x-2 \geq 0 \\ x \geq 2 \end{array} \quad \text{when} \quad \begin{array}{l} x+3 \geq 0 \\ x \geq -3 \end{array}$$

$$\text{b) } x(x-5) > 0 \quad \text{when} \quad \begin{array}{l} x > 0 \\ x > 0 \end{array} \quad \text{when} \quad \begin{array}{l} x-5 > 0 \\ x > 5 \end{array}$$

6) Sketch the graph of regions:

$$\text{a) } Y \leq x^2 + x - 12$$

$$\text{When } x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

$$X = -4 \text{ or } 3$$

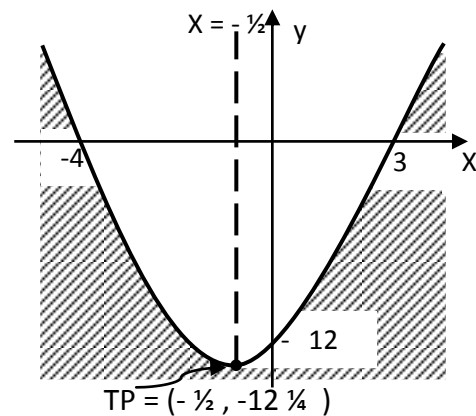
$$y\text{-intercept} = c = -12$$

$$\text{Test point: } (1, 1)$$

$$1 \leq 1^2 + 1 - 12$$

$$1 \leq 1 + 1 - 12$$

$$1 \leq -10 \quad \text{False}$$



$$\text{c) } Y > 2x^2 - 2x - 4$$

$$\text{When } 2x^2 - 2x - 4 = 0$$

$$2(x^2 - x - 2) = 0$$

$$2(x+1)(x-2) = 0$$

$$x = -1 \text{ or } 2$$

$$y\text{-intercept} = c = -4$$

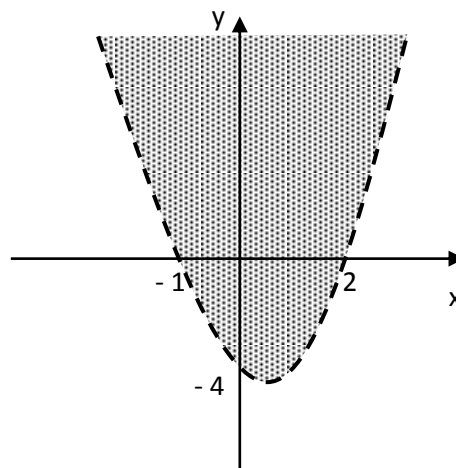
$$\text{Test Point } (1, 2)$$

$$Y > 2x^2 - 2x - 4$$

$$2 > 2 \cdot 1^2 - 2 \cdot 1 - 4$$

$$2 > 2 - 2 - 4$$

$$2 > -4 \quad \text{True.}$$





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